

TEST PAPERS
IN
ELEMENTARY ALGEBRA
DURELL



& M & Co

M/
1-2

5

FRANKS (M) LIBRARY

Blore-42

Accession No :

312

U.D.C. No :

512/DUR

Date :

22.1.81

N14

Acc/m/331

24/1/76

R.B.A.N. M'S HIGH SCHOOL

Bangalore.

~~May 331~~

~~Acc. m / 331
24/1/76~~

TEST PAPERS IN ELEMENTARY ALGEBRA

R.B.A.N.M'S H.S (M) LIBRARY

Blore-42



312.

512/ D.V.R / N / 4

22-1-81



MACMILLAN AND CO., LIMITED

LONDON • BOMBAY • CALCUTTA
MELBOURNE

THE MACMILLAN COMPANY

NEW YORK • BOSTON • CHICAGO
DALLAS • SAN FRANCISCO

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

TEST PAPERS

IN

ELEMENTARY ALGEBRA

~~Al: 7~~
BY

CLEMENT V. DURELL, M.A.

ASSISTANT MASTER AT WINCHESTER COLLEGE

R.B.A.N.M'S H.S (M) LIBRARY

Blore-42

Accession No ;

312.

U.B.N.M'S H.S (M) LIBRARY
No :

512 / DUR / NI

Date ;

22-1-81



MACMILLAN AND CO., LIMITED
ST. MARTIN'S STREET, LONDON

1914

R.B.A.N.M.'S (M) LIBRARY

Plate-42

Accession No :

312

U.D.C. No :

512/012

Date;

22.1.81

N14

~~1489~~

~~B2~~

~~N14~~

COPYRIGHT



~~Acc/m/333~~
~~24/1/76~~



~~m/331~~

PREFACE

THIS collection of papers, designed primarily for out-of-school work, is intended to supplement those given in existing text-books. Many questions are included which are unsuitable for the non-specialist, but it is hoped that the papers are sufficiently long and sufficiently numerous to permit of a selection in such cases. Few graphical questions have been inserted in the papers, in view of the fact that it is often preferred to restrict this work to the class-room; a set of graphical examples is, however, added at the end for those who think otherwise.

Answers are not given to questions which do not require any intermediate work, as, for example, in the case of ordinary factorisation. Such answers are of no use to the master, and destroy the utility of the book for out-of-school work.

I am indebted to the following authorities for permission to use questions taken from examination papers: The Syndics of the Cambridge University Press, the Clarendon Press, the University of Cambridge Local Examinations and Lectures Syndicate, the Delegates of Local Examinations University of Oxford, the Oxford and Cambridge Schools Examination Board and the Controller of His Majesty's Stationery Office. Free use has also been made of papers set at Winchester College.

WINCHESTER, *December*, 1913.

R.B.A.N. M'S HIGH SCHOOL,

Bangalore.



R.B.A.N. M'S HIGH SCHOOL

Bangalore.



CONTENTS

PAPERS		PAGE
1-10.	Generalised arithmetic ; substitution ; meaning of symbols ; addition ; subtraction ; multiplication and division by single terms ; brackets - -	1
11-20.	Simple equations ; problems ; fractions with numerical denominators - - - - -	11
21-30.	H.C.F. and L.C.M. of single terms ; multiplication and division ; simultaneous equations ; graphs of simple functions ; fractions with monomial denominators - - - - -	21
31-40.	Easy factors ; graphical solution of simultaneous equations ; quadratic graphs - - - - -	31
41-50.	Fractions ; harder factors ; H.C.F. and L.C.M. by factors ; simultaneous equations in three unknowns	42
51-60.	Quadratics by factors - - - - -	52
61-70.	Quadratics by completing the square ; functional notation ; remainder theorem - - - - -	63
71-80.	Detached coefficients ; very simple literals - -	72
81-90.	Simultaneous equations, one being quadratic ; undetermined coefficients - - - - -	83
91-100.	Square root ; harder literals ; H.C.F. by rule -	93
101-110.	Simple indices ; variation ; easy ratio - - -	102
111-120.	Logarithms ; simple gradients - - - - -	112
121-130.	Ratio and proportion ; indices and surds ; easy arithmetical progressions - - - - -	122

PAPERS	PAGES
131-140. Simultaneous quadratics ; arithmetical and geometrical progressions - - - - -	1
141-150. Elementary theory of quadratics ; harmonical progressions - - - - -	1
151-160. Permutations and combinations - - - - -	1
161-170. Partial fractions ; easy probability - - - - -	1
171-180. Practice in manipulation ; hard miscellaneous equations ; logarithms to any base ; maxima and minima - - - - -	1
181-190. Algebraic form ; symmetry ; imaginaries ; limitation of values of a function ; harder graphs - - - - -	1
191-200. Miscellaneous series ; induction ; binomial theorem ; indeterminate equations - - - - -	1
GRAPHICAL EXAMPLES - - - - -	1
ANSWERS - - - - -	2



TEST PAPERS IN ELEMENTARY ALGEBRA.

PAPERS 1-10 include

Generalised arithmetic; substitution; meaning of symbols; addition; subtraction; multiplication and division by single terms; brackets.

1.

1. If $a=5$, what is the value of

(1) $a+4$; (2) $4+a$; (3) $4a$; (4) $6a-6$; (5) $3-2a$?

2. A man walks x miles in the morning, y miles in the afternoon, and z miles in the evening; how far does he walk in the whole day?

3. If $x=2$, $y=3$, find the value of (1) $2x+3y$; (2) x^2+xy ;

(3) $3x^2-5xy+2y^2$; (4) $\frac{x}{y}+\frac{y}{x}$; (5) $5x^2y^2$; (6) $7xy^2$.

4. (1) Find the sum of $2a-3b+c$; $5b-3a-2c$; $8c-2b-4a$.

(2) Subtract $6a^2+3ab-2b^2$ from $3a^2-5ab+8b^2$.

5. Simplify (1) $a-2(b+c)$.

(2) $3(2a-b)-2(3a+b)$.

(3) $2a(b-c)-3b(2a+c)+4c(a-2b)$.

(4) $2a-[7-4(a-2\{1-3a\}-2)+a]$.

6. At a meeting there are seats for $3x+2y-5z$ people; if $2x-y$ men and $3y-2z$ women are present, find the number of empty places.

2 TEST PAPERS IN ELEMENTARY ALGEBRA

7. Add five times the sum of a and b to the excess of tl times a over two times b .

8. Find the values of $3x^2 - 5x + 2$ when x has the values 3, 2, 1, 0, -1, -2, -3.

2.

1. If $a = -1$, $b = 2$, $c = 0$, what is the value of

- (1) $2a$; (2) a^2 ; (3) $a + b$; (4) ab ; (5) abc ;
(6) $a^2 + b^2 + bc$; (7) $3a^2 - 5ab + 6b^3$?

2. A man bicycles 12 miles an hour; how many minutes will he take to go (1) 3 miles, (2) x miles; and how far will he go in (1) 10 minutes, (2) y minutes?

3. (1) Add together

$$7a - 3b + 2c; \quad 3a + b - 11c; \quad -2c + 3b - 9a.$$

(2) Subtract $8a + 9b$ from a .

4. (1) The temperature rises 10 degrees, then falls 5 degrees, then rises 3 degrees; what is (1) the total rise, (2) the total fall in temperature?

(2) A man walks x miles due East, then y miles due West, then z miles due East; how many miles (1) due East, (2) due West is he of his starting point?

5. By how much is (1) 5 less than 8, (2) -5 less than 8, (3) x less than 8, (4) x less than y ? What must be added to $7x - 2y + 3z$ to obtain $2x + y - 11z$?

6. Simplify (1) $2a(3 - b) - 3b(2 - a) - a + 7(b - 2a) + 10$.

(2) $a(b - c) + b(c - a) + c(a - b)$.

(3) $- \{3a - 2[2a + 3(4 - a) - 9] + 1\}$.

7. (1) Multiply $3x^2 - 5x - 2$ by $2x$.

(2) Divide $9x^3 - 15x^2 + 3x$ by $3x$.

(1) What value of x makes $3x + 1$ equal to $2x + 5$?

(2) If $3x$ is equal to -9 , what are the values of
 $2x$; x^2 ; $-x^2$; $(-x)^2$; $(x-1)^2$?

3.

1. If $a = 4$, $b = -2$, $c = 0$, $d = 3$, find the values of

(1) a^2 . (2) $2a$. (3) $abc + b^2d - ab^2$.

(4) $\frac{a}{b} - \frac{b}{d} + \frac{1}{3}ab$. (5) $a^2(b-c) + b^2(c-d)$.

2. Explain the difference between (1) $3a$ and a^3 ; (2) ab^2 and $(ab)^2$; and state fully why $x^2 \times x^3 = x^5$; $(x^2)^3 = x^6$; $x^5 \div x^2 = x^3$.

3. A man walks 3 miles an hour uphill and 4 miles an hour downhill. He starts by going 12 miles uphill, then x miles downhill, then y miles uphill; how long does he take altogether?

4. The sum of two numbers is 50; find one of them if the other is (1) 18; (2) x ; (3) 65; (4) four times the first.

5. A man has $x\text{£}$; find how many pence he has left if he spends (1) 3£ ; (2) 7s.; (3) y pence; (4) y florins; (5) $7x$ shillings.

6. Simplify

(1) $x^2(3x - 2y) - 2x(3y^2 - x) + 5xy(2x - y)$.

(2) $2 - 3\{1 - 2[2 - 4(1 - 5\overline{2 - 3} - 4) + 1] + 1\} - 1$.

(3) $3x(y - y[2x - y] + x) - 3y(x - x[2y - x] + y)$.

7. (1) From the sum of $x - 2y + 3z$ and $2y - 7z$ subtract the sum of $2y - x - z$ and $3x - 2z$.

(2) Divide the product of $5a^3 - 15a^2b + 20ab^2$ and $6ab$ by $15a^2$.

8. A man is 45 years old; how long is it since his age was (1) 32; (2) x ; (3) $15 - y$; (4) $45 - x$? and when will his age be (1) 57; (2) x ; (3) $15 + x$; (4) $45x$?

4 TEST PAPERS IN ELEMENTARY ALGEBRA

4.

1. Simplify

- (1) $(-a)(2b)$. (2) $(-2a)(-b)$. (3) $(-3a^2b)(4ab^3)$
 (4) $(5a^2bc)(-3ab^3c^3)$. (5) $(-3a^2bc^3)^4$. (6) $(-2ab)^2(3a^2b)$
 (7) $15a^4b^4 \div 3ab$. (8) $6a^6b^6 \div 3a^3b^3$. (9) $10a^4b^2 \div (-2a^2b)$
 (10) $-16x^3y^3 \div (-8xy^2)$.

2. If $y = 3 - 2x - x^2$, find the values of y when x is successively equal to 3, 2, 1, 0, -1, -2, -3.

3. If $x = 2$, $y = 3$, $z = -1$, find the values of

- (1) xy . (2) x^y . (3) y^x . (4) $(y+z)^x$.
 (5) $y^3 + z^3 - x^3 + 3xyz$. (6) $\frac{x}{y+z} + \frac{z}{x-y}$.

4. A man buys x lbs. of tea at 2s. a lb. and y lbs. of coffee at 1s. 6d. a lb. and pays a sovereign; how much change does he get (1) in pence, (2) in shillings?

5. Simplify

- (1) $3\{2x - 4(2 - x - 3[1 + x] - 1) - x\} - 2[x - (5 - x)]$.
 (2) $8a - [3b - 2(a + b - c) - 3(c - a)]$.
 (3) $(-x)(-3) + (-1)(1 - x) - x^2$.
 (4) $(3x)^2 - (-3x)^2 + (7x)(-2x)$.

6. If $a = 2x - y - z$, $b = x - 3y$, $c = y - z$, and if $x = -1$, $y = 2$, $z = 1$, find the values of

- (1) $abc + ab + c$. (2) $\frac{a+b}{c}$. (3) $2a^2 - b^2 + c^2$.

7. Find the sum of

$3x^3 - 2x + 7x^2 - 5$; $2x^2 - 3 - x^3$; $4x^2 - 8x + 11$; $3x^2 - 2x^3 + 7x - 5$ and subtract $2x^3$ from $3x^2$.

8. A has 5£ more than B and B has 4£ more than C ; B has x £, how much have A and C together? If now A gives B 2£ and C gives B 6£, how much has A more than C ? If B has now twice as much as C , what is the value of x ?

5.

1. If $a=2$, $b=-\frac{1}{2}$, $c=-1$, find the values of

$$(1) \frac{a}{b}, \quad (2) abc, \quad (3) a^2 + ab - c^2, \quad (4) \frac{a+b}{a-c}.$$

$$(5) b^a, \quad (6) 2a(b-c) - b(2c-a).$$

2. A man leaves his property amounting to $7a - 3b + 2c$ pounds to his three sons; the eldest gets $6a - 2c$ pounds and the youngest gets half that amount; how much does the other son inherit?

3. A man walks $2x - y$ miles due East, then $x + 2y$ miles due West; how many miles (1) due East, (2) due West is he of his starting point?

4. If $a=2$, $b=1$, $c=-2$, prove that

$$a(b+c) + b(c+a) + ac = a(b-c) + b(c-a) - ac^2,$$

and that $a^2 - b^2 + c^2 + 2ac = (a+b+c)(a-b+c).$

5. Simplify

$$(1) \frac{1}{2}(a-3b) - \frac{1}{4}(3a+b).$$

$$(2) a - [2a - (a - \{3a - \overline{a+a}\} - a)].$$

$$(3) x^2(y^2 - z^2) - 2y^2z^2 + 3y^2(x^2 - z^2) + 2z^2(x^2 - y^2) - x^2.$$

6. (1) If $3x=10+x$ and $5y=12-y$, find the values of $x+y$ and xy .

(2) If $a=2x-y$, $b=x-3y$, $c=7x+y$, express $2a-b+3c$ in terms of x and y , and find the value of

$$ab - b^2c + 4ac \text{ if } x=1, \quad y=2.$$

7. (1) From the sum of $2x(3x-y)$ and $3y(3x+y)$ subtract $4x^2 + 8xy - y^2$.

(2) Multiply $x^3 - 2x^2 - 3x + 4$ by x^2 ; $-4x$; -3 , and add the three results together.

6 TEST PAPERS IN ELEMENTARY ALGEBRA

8. A man buys x bananas for a pound and sells them for a penny each; what is his profit in pence? If his profit is half-a-crown, what is the value of x ?

6.

1. Simplify

$$(1) 5ax \times 2bx \times 3abc.$$

$$(2) 2x \times (-3y) \times (-y).$$

$$(3) 4x^2 \times 2y^2 \div (-xy).$$

$$(4) (2x^2y)^3 \div (-3xy)^2.$$

$$(5) \frac{1}{2}x - \frac{1}{3}x.$$

$$(6) \frac{x}{5} - \frac{2x}{15}.$$

$$(7) x - \frac{1}{2}x.$$

2. By how much is (1) 54 larger than 29; (2) 54 larger than x ; (3) y larger than 29; (4) a larger than b ; (5) 54 less than 71; (6) 54 less than a ; (7) x less than y ?

3. (1) If $x = 2y$ and $x + y = 18$, find the value of $2x - 3y$.

(2) If $x = 2y - 3$ and $y = 3z + 1$, prove that $x + 1 = 6z$.

4. Subtract $2x^3 - 3x^2 - 5x + 11$ from $x^3 + 4x^2 - 7x + 9$, and add $2x(3x^2 - 5x + 4)$ to $3(3x^2 - 7x - 1)$.

5. (1) Find the sum of four consecutive odd numbers, the smallest of which is x .

(2) Write down seven consecutive odd numbers, of which x is the middle number.

6. Simplify (1) $2a - 3(2b - c - 4[a - b + 3c - a]) - a$.

$$(2) 2x(3x + y - z) - 3y(2x - y - 2z) + z(x + y - z).$$

$$(3) 1 - 2(1 - 2[1 - 2\overline{1 - 2}] + 1) - 2.$$

7. If $a = 3$, $b = -2$, $c = \frac{1}{2}$, $d = 0$, find the values of

$$(1) a^2 + b^2 + c^2.$$

$$(2) ab + d(a + b) - bc + ab^2.$$

$$(3) (a + 2b)(b + bc).$$

$$(4) \frac{a + b + c}{a - b + c} + \frac{a + d}{a - d}.$$

TEST PAPERS IN ELEMENTARY ALGEBRA 7

8. (1) If x is the smallest prime number larger than 20, find the value of x .

(2) If x and y are consecutive prime numbers, and if their difference is 6, find the least possible value of x .

7.

1. If $x = \frac{1}{2}$, $y = \frac{1}{3}$, $z = 3$, $a = -4$, $b = 5$, find the values of

$$(1) x + y - xy. \quad (2) z(x + y) - a(x - y). \quad (3) x^2 - yz - \frac{a}{b}.$$

$$(4) \frac{2a + b + z}{a + b + z} - \frac{(a + z)(z + b) + (a + b)(z + a)}{(ax + yz)(a + b) - (a^2 - bz)(b^2 + 2az)}.$$

2. A man is x years old and his wife is y years old : (1) what is the age of each in 10 years' time, in z years' time, b years ago ; (2) what is the sum of their ages in 8 years' time, in z years' time ; (3) how long will it be before the man is 60 years old, z years old ; (4) when will the woman be x years old ?

3. If $a = 2x - y + z$, $b = 3x - 2y - 3z$, $c = 2y - 4x + 2z$, express $3a - 2b + 3c$ in terms of x , y , z , and find the value of

$$a^2 - b^2 - c^2 - 2bc \text{ if } x = -1, \quad y = -2, \quad z = 3.$$

4. (1) If x oranges cost one shilling, how much do twenty oranges cost ?

(2) If x oranges for a shilling is the same rate as y pence per dozen oranges, prove that $xy = 144$.

5. Find the sum of

$$2a(3b - c + a) ; \quad a^2 - 7ab + 5ac ; \quad 3ab - a(2a + c) ;$$

and subtract $9a^2 + 5ab$ from $10b^2 - 3ab$.

6. Simplify (1) $2a - \{b - 3(c - a + 2[b - c] - b) + a\}$.

$$(2) 2a + 3\{2 - \overline{a + b}\} - a\{3 - 2[a + 1]\}.$$

8 TEST PAPERS IN ELEMENTARY ALGEBRA

7. In the series of numbers 1, 3, 5, 7, 9, ..., what is the value of (1) the 8th number, (2) the 50th number, (3) the x^{th} number. It can be proved that the first x numbers add up to x^2 ; verify this for $x=3, 4, 5, 6$; what is the sum of the first $3x$ numbers?

8. What must be added to x to make y ? What must be subtracted from x to leave y ? By what must x be multiplied to give y ? By what must x be divided to give y ? Verify each of your answers by putting $x=12, y=4$.

8.

1. If $x=1, y=4, z=2, w=0$, find the values of

(1) $\sqrt{x+2y}$. (2) $\sqrt[3]{2y^2+3z^2+3xy+yz}$.

(3) $\frac{x}{yz} + \frac{w}{xy} - \frac{z}{xy}$. (4) $(x-y)^2 + (z-w)^2 - 2(x-w)(y-z)$.

2. Find the sum of $3x-2x^3-4x^2+2$; $7x^2-1+6x^3-3x$; $8-x-3x^3+2x^2$; x^3-11 ; $x-x^2$; x^3-x+1 .

3. (1) If ten men can mow a field in 3 days, how long will 7 men take?

(2) If x men can mow a field in y days, how long will z men take, and how many men will be required to mow it in q days?

4. (1) Subtract $3x-x^2$ from the result of subtracting $1-4x^3$ from $7x^2-2x+3x^3-5$.

(2) By how much does $x^2-11xy-2y^2$ exceed the sum of $3xy-2x^2+y^2$ and $4y^2-5xy+x^2$?

5. Simplify

(1) $3[a-2(b-3a)+3(a+1+2b)-4(2a-b-5)-14]$.

(2) $\frac{1}{3}[a-\frac{1}{2}(3b-a)+\frac{1}{4}(2a-b)]$.

(3) $(-16x^4y^5+8x^3y^6-12x^5y^4) \div (-4x^3y^3)$.

6. Find the integral values of x between $+4$ and -4 for which the expression $x^3 - 3x^2 - 10x + 24$ vanishes.

7. If x and y are two numbers, x being the larger, express by an equation the following statement: the difference of the square of their sum and the square of their difference is equal to four times their product. Verify this if $x = 8$ and $y = 3$.

8. If $3x = a - 3b - 2c$, $2y = 2a + b - c$, $z = 3a - 2b + 3c$, express (1) $x + y + z$; (2) $6x + y - \frac{1}{2}z$ in terms of a , b , c . If $a = 4$, $b = 1$, $c = -1$, find the value of $\frac{2x + y}{z}$.

9.

1. If $a = 2$, $b = -2$, $c = 0$, $d = -1$, find the values of

$$(1) a^2 + b^2 + (c - d)^2. \quad (2) (a - b)(b - c)(c - d)(d - a).$$

$$(3) (a - d)^a + d^a. \quad (4) a^3 + b^3 + c^3 - 3abc.$$

$$(5) \sqrt{3(a^2 + d^2) - b(2a - d)}.$$

2. Find the sum of $3a - 2b + c - 7d$, $2a - 4c$, $3b - 2a - c + d$, $7d - c + a$, $4c - 2b - a - 5d$; and of $\frac{1}{2}x - \frac{1}{3}y - z$, $\frac{1}{4}x + 2y + \frac{1}{3}z$, $x - \frac{1}{2}y - \frac{1}{2}z$; $\frac{3}{4}z - \frac{2}{3}y$.

3. Through how many degrees does (1) the minute hand, (2) the hour hand of a clock turn in 1 hour, 15 minutes, 1 minute, x minutes? What is the angle between the two hands at 10 minutes past twelve, x minutes past twelve, 10 minutes past one, x minutes past one?

4. Simplify

$$(1) a - [2a - 3(a - 2\{a - \overline{a - 2a}\} + 2a) - a].$$

$$(2) a - [2a + 3(b - a)] + b - [2b + 3(a - b)].$$

$$(3) 2x(3y - 2x) - 2y(x - 4y) - 4(x^2 + y^2).$$

$$(4) (-2xy)(-3x^2). \quad (5) (-3x)^3 + 3x^3.$$

$$(6) 12x^4y^4 \div (-6xy^2).$$

10 TEST PAPERS IN ELEMENTARY ALGEBRA

5. Find the sum of the products of $7x^4 - 2x^2 - x + 2$ by x^3 , $-2x^2$, $3x$, -2 .

6. Express the following facts in words :

$$(1) (2x)^2 = 4x^2. \quad (2) (x+y)(x-y) = x^2 - y^2.$$

$$(3) (x+y)^2 = x^2 + y^2 + 2xy.$$

7. (1) The difference of two numbers is 27, the smaller is 11; what is the other? (2) The difference of two numbers is x , the smaller is a ; what is the other? (3) The sum of two numbers is y , one of them is b ; what is the other?

8. If a train travels x miles an hour, how many feet does it go in one second?

10.

1. If $y = \frac{x^2 - 3x + 2}{x - 6}$, find the values of y when

$$x = -3, -2, -1, 0, 1, 2, 3, 4, 5.$$

2. If $a = 2b - 1$, $b = 3c - 2$, $c = 4d - 3$, $3d = 6$, find the values of $a + b + c + d$; $\frac{a}{c} + \frac{b-5}{d}$; $(2b-a)^2 + (c-3d)^3$.

3. Simplify

$$(1) (-1)^3 + (-2)(-3)^2 - (-2)^4 + 3(-1)(-2)(-3).$$

$$(2) (2ab^2)^3 \times (-3ab)^3 \div (-9a^2b).$$

$$(3) [3a + b - (8b - 11a)] \div 7.$$

4. A man's salary is x £ for his first year, and it increases by y £ every year; what is his salary for his 3rd, 5th, 10th, 30th, n th year?

5. From the sum of

$$2a - 3b - c, \quad 4a - 7d, \quad 2b + 3c - a, \quad 3c - 2d + 4a, \quad 2b - 5c,$$

subtract $2(a - b + 3c) - 3(b - 2a - c).$

TEST PAPERS IN ELEMENTARY ALGEBRA 11

6. Start with the number x and perform the following operations: double it, square the result, then multiply by -3 and add 9, then divide by -3 and add 15, divide the result by 4 and subtract 3. What is the answer?

7. A man buys x horses for y £ each; he sells p of them for a £ each and q of them for b £ each and the rest for c £ each; how much does he gain?

8. (1) Find which of the values $x = +2, -2, +4, -4$ make $x^4 - 2x^3$ equal to $7x^2 + 2x + 8$.

(2) If $x^3 - 7x^2$ is equal to $2x + a$ when x equals -1 , find the value of a .

PAPERS 11-20 include

Simple equations; problems; fractions with numerical denominators.

11.

1. If $a = 4, b = 2, c = -2, d = 0$, find the values of

(1) $3a + 4b - 2c + 3(a + b + c)$. (2) $a(b - c)(b + c) + d(a^3 - b^3)$.

(3) $bc^2 - (bc)^2 - ac$. (4) $(ab + cd)(ac + bd)$.

2. Simplify

(1) $2x + y - 3x - y + xy$. (2) $2x + y - 3(x - y) + xy$.

(3) $2\{x + y - 3(x - y)\} + xy$. (4) $\frac{35a^2b}{28ac} \div \frac{5b}{5c}$.

(5) $\frac{x}{2} + \frac{x}{3} - \frac{x}{5}$.

3. Find what value of x makes $4(x - 5) - 4$ equal to $2(x - 2) + 6$, and check your answer.

12 TEST PAPERS IN ELEMENTARY ALGEBRA

4. In 15 years' time a man will be twice as old as he was 7 years ago ; how old is he now ?

5. Simplify

$$(1) 3x\{2y - 4(x - 7y + 3\overline{y + 2x}) + x\}.$$

$$(2) 2a(b - c) + 3b(c - a) - 2[c(a + b) - 3b(a + c)].$$

6. If $2x - 3 = 7 - 3x$ and $6 - 4y = 7y + 28$, find the value of

$$(1) x + y; \quad (2) x^2 + y^2.$$

7. Multiply $a^2 + b^2 + c^2 - ab - bc - ca$ by a , also by b , and by c , and find the sum of the three results.

8. Find three consecutive even whole numbers, such that three times the largest is equal to twice the sum of the other two.

12.

1. If $a = 1$, $b = 2$, $c = -2$, find the values of

$$(1) (a + b)^2 - a^2 - b^2; \quad (2) (b - c)^2 - (b^2 - c^2); \quad (3) (a - c)(c - b).$$

2. Add together $7a - 4b - c$; $3c + 2b - a$; $b - c - a$; and subtract $7x^2 + 3x + 5$ from $7x^3 + 3x^2 + 5x$.

3. Find the value of x which makes the sum of $2(x - 3)$ and $3(1 - x)$ equal to $x + 1$. Check your answer.

4. If ax^2y is equal to 60 when $x = 2$ and $y = 5$, find its value when $x = 5$ and $y = 2$.

5. Simplify

$$(1) 5x^2 \times 2x^3. \quad (2) \frac{15a^2bc^3}{25ab^2c}. \quad (3) \frac{x}{2} - \frac{x}{3}.$$

$$(4) \frac{x - 1}{2} - \frac{2x - 1}{4}. \quad (5) \frac{x + 1}{3} - \frac{x + 1}{2} + \frac{2 - x}{4}.$$

6. C is a point on the line AB , AC equals $3x - 2y$ inches, and AB equals $x + y$ inches ; what is the length of CB ? If D

TEST PAPERS IN ELEMENTARY ALGEBRA 13

is a point on the line such that $AC = DB$, find the length of CD .

7. A has $x\text{£}$; B has 12£ more than twice what A has; C has 11£ more than A and 9£ less than B . What is the value of x ?

8. What is the area of (1) the floor, (2) the four walls of a room which is firstly 18 feet long, 12 feet wide, 10 feet high, and secondly x feet long, y feet wide, z feet high?

13.

1. Add together $\frac{1}{2}x - \frac{1}{3}y$; $\frac{1}{3}x + \frac{1}{2}y$; $\frac{3}{4}y - \frac{2}{3}x$, and multiply the result by $60xy$.

2. Simplify

$$(1) 2a(b-c) - 3b(c-a) + 2c(2a-b). \quad (2) 5x^2 \times (-2xy).$$

$$(3) (-3x)^2 \times (2x)^3. \quad (4) (-6xy)^3 \div (-2xy).$$

3. A man receives x pounds y shillings z pence, and gives away $y-z$ pounds $2x-y$ shillings $x+y$ pence; how many farthings has he left?

4. Solve the equations:

$$(1) 2x - 3 = 21 - 6x, \quad (2) \frac{2x}{3} - \frac{x}{4} = 20,$$

$$(3) 4(x-1) - 3(x-2) = 5(3-x) - 1,$$

and check each answer.

5. A walks x miles in y hours, B walks 1 mile per hour slower than A . How far does B walk in y hours?

6. From the formula $V = \frac{2}{21}r^2h$,

$$(1) \text{ find } V \text{ when } r = 3\frac{1}{2}, h = 12$$

$$(2) \text{ find } r \text{ when } V = 84, h = 5\frac{1}{2}$$



14 TEST PAPERS IN ELEMENTARY ALGEBRA

7. A is twice the age of B ; in 14 years' time their united ages will be five times B 's present age. Find their present ages.

8. (1) Find which, if any, of the values $x = \pm 2$ or ± 4 satisfy the equation $x^4 - 2x^3 = 7x^2 + 2x + 8$.

(2) Prove that $x = 2$, $y = -1$ and $x = \frac{1}{7}$, $y = \frac{19}{7}$ are solutions of each of the equations $2x + y = 3$; $x^2 - xy + y^2 = 7$.

14.

1. If $x = 2a - 3b + 4c$, $y = a - 2b + 3c$, $z = 3a - b - c$, express $3x - 5y + 2z$ in terms of a , b , c . Also, if $a = 1$, $b = 1$, $c = 2$, find the value of $xy + yz + zx - y^2 - z^2$.

2. For what value of a does the sum of $2(3a - 1)$ and $3(1 + 2a)$ amount to $4(2a + 3)$. Check your answer.

3. Simplify (1) $\frac{2a}{3} - \frac{3a}{5} + a$. (2) $\frac{a + 2b}{4} - \frac{a - b}{20} + \frac{2a}{5}$.

(3) $\frac{2a^2}{7b^2} \times \frac{3ab}{4a^4} \times 5$.

4. Solve the equations:

(1) $5(x - 2) = 3(x - 1) - 1$. (2) $\frac{x}{2} + \frac{x}{3} = 5$.

(3) $\frac{1}{3}(4x + 1) - \frac{1}{4}(2 - 3x) = \frac{1}{6}(26 - x)$.

5. A boat goes at the rate of x miles per hour when the crew row m strokes per minute. How many yards do they cover between each stroke? How many strokes will they row in a course of y miles? [If you cannot do this question as it is here set, put $x = 5$, $m = 30$, $y = 4$, and then work it out.]

6. A has 8£ more than B and B has 10£ more than C ; they have 178£ between them. How much has A ?

7. If sugar costs x farthings per lb., find in £ the price of a ton.

8. What number is doubled by adding 20 to it and dividing the result by 3? What number is halved by adding 20 to it and dividing the result by 3?

15.

1. Simplify

$$(1) \ 5x - (3 + x) - 2[x - (1 - x) - 3x].$$

$$(2) \ -3x^2 \times 2xy \times (-xy^2)^2. \quad (3) \ (-2ab)^3 \div 2a^2.$$

2. The population of a town at the beginning of a year is $3a - b + 2c$; during the year there are $2c - a$ deaths and $b + 3c - 2a$ births, also $a - b$ people come into the town and $3a + b$ people leave it. What is the population at the end of the year?

3. Solve the equations:

$$(1) \ \frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x = x + 1. \quad (2) \ 2(x - 1) - 3(3 + x) = 5 + x.$$

$$(3) \ \frac{2x - 1}{3} - \frac{3x + 1}{4} = x. \quad (4) \ \frac{5}{x} + \frac{1}{3} = \frac{3}{4}.$$

4. A gives B x miles' start; A walks y miles an hour and B walks z miles an hour; how long will A take to catch up B ? [If you cannot do this question as it stands, put $y = 4$ and $z = 3$, and then do it.]

5. A is 10 years old and B is 31 years old; in how many years' time will A be half the age of B , and how long ago was B four times the age of A ?

6. A man walks x miles an hour; how many feet does he go in a second? If x miles an hour is the same pace as one chain in y minutes, prove that $4xy = 3$.

16 TEST PAPERS IN ELEMENTARY ALGEBRA

7. Four times a number is as much below 300 as six times that number is above 300 ; find the number.

8. If $x = 3 - 2z$ and $y = 3z - 2$, prove that $3x + 2y = 5$ and that $2x + 3y - 5z = 0$. Also, if $x = z + 12$, find the value of y .

16.

1. If $a = -2$, $b = 3$, $c = 2$, $d = 0$, find the values of

$$(1) (a + b) \cdot (b + c) \cdot (c + d)^2. \quad (2) (a - b)^2 + (b - c)^2 + (c - a)^2.$$

$$(3) a + 2b(3 - cd - a\{2b - ad\} - b^2). \quad (4) \frac{a}{b+c} - \frac{c}{a+d}.$$

2. Simplify (1) $\frac{2}{3}x - \frac{1}{4}x + \frac{1}{2}(1 - x) - 2$.

$$(2) \frac{2x-1}{3} + \frac{2y+1}{5} - \frac{3x-2y}{10} - 1.$$

$$(3) \frac{5}{2x} - \frac{3}{5x} + 2 - \frac{1}{x}.$$

3. Two consecutive odd numbers are such that three times the smaller is greater than twice the larger by 37 ; find the numbers.

4. Solve the equations :

$$(1) 2x - (1 - 2x) = 5 - 3(1 + x).$$

$$(2) \frac{x-1}{2} + \frac{x-3}{4} = 2.$$

5. A man buys 5 horses for x £ each and 3 horses for $2x - 15$ £ each ; he sells two of them for $x + 1$ £ each and the remainder for $85 - 3x$ £ each ; what profit does he make ?

6. A particle moves s feet in t seconds, where s and t are always connected by the relation $s = 200t - 16t^2$; find the distance it has gone after 1, 2, 3 seconds, and find the distance it goes in the 5th second.

7. State a problem the solution of which leads to the following equation :

$$2(x - 30) = \frac{x}{2} + 3.$$

Then solve it.

8. A man walks from X to Y at 4 miles an hour, rests for two hours, and then walks back again at 3 miles an hour. If his walk lasts nine hours altogether, how far is Y from X ?

17.

1. If $a = \frac{1}{2}$, $b = \frac{1}{3}$, $c = -1$, find the values of

$$(1) ab + bc + ca. \quad (2) (a - c)(b - c)^2. \quad (3) \frac{a+b}{a-b} + \frac{b-c}{b+c}.$$

2. Simplify (1) $\frac{2x}{3y} - \frac{3x}{2y} + \frac{4y}{3x} - \frac{3y}{4x}.$

(2) $x - 2[1 - x(2 - x) - 3(x - 1)] + 7x(2 - x).$

(3) $\frac{2x - 3y}{5} - \frac{3x + y}{2} + 2x + \frac{1}{3}y.$

3. $ABCD$ is a straight line and $AB = BC$; E is the middle point of CD . If $AD = 2x - 3y$ inches and $AB = x - 4y$ inches, find the lengths of CD and BE .

4. If the sum of two numbers is equal to twice their difference, prove that twice the greater exceeds the smaller by the sum of the greater and twice the smaller number.

5. Solve the equations :

(1) $2(x - 1) + 4 = 5 - 3(2 - 7x).$

(2) $\frac{x}{3} - 8 + \frac{x + 40}{5} = 47 - \frac{x}{4}.$

6. A man receives, as change for a five-pound note, x sovereigns, $2x + 6$ half-crowns and $3x + 1$ shillings. Find x .

18 TEST PAPERS IN ELEMENTARY ALGEBRA

7. The perimeter of a rectangular field is $11x - 2y + 3z$ yards, and its length is $5x - 2z$ yards. Find its breadth.

8. A , B , C start with equal sums of money; C receives 3£ from A and 4£ from B , and then has 2£ more than A and B together have. How much had each at first?

18.

1. (1) Add together

$$(2a - 3b)x - (a + b)y; \quad (3x - y)a + (2x + y)b; \\ (5a - 2x)y - (3a - x)b.$$

(2) Subtract

$$4(a - 2b - c) - 3(2b - a + c) \text{ from } 2(3c - a) - 3(2a + b).$$

2. Find the value of $\frac{x^2 + 2xy + y^2 - z^2}{(x + y - z)(x + y + z)}$ when

$$(1) \ x = 2, \ y = 1, \ z = -1. \quad (2) \ x = 0, \ y = 2, \ z = 1.$$

$$(3) \ x = \frac{1}{2}, \ y = \frac{1}{3}, \ z = \frac{2}{3}.$$

3. The perimeter of a triangle is 3 feet; the greatest side is 9 inches longer than the least side, and the sum of the two smaller sides is 2 inches greater than the largest side. Find the shortest side.

4. A man buys x eggs at the rate of 8 for a shilling and y eggs at the rate of 9 for a shilling, and sells them all at 6 for a shilling. How many pence profit does he make?

5. Solve the equations:

$$(1) \ x - 5(1 - x) - 3(1 + 2x) = 4 - 3(2x - 1) + 5(1 + x).$$

$$(2) \ \frac{2x - 1}{5} = \frac{1 - 3x}{4} + 3. \quad (3) \ \frac{3}{2x} - 4 = 3 - \frac{9}{x}.$$

6. In ten years' time I shall be five times as old as I was eighteen years ago. What is my age now?

7. If x shillings and $x + 7$ half-crowns make up two guineas, what is the value of x ?

8. If the price of x £ a ton is the same as y farthings a lb., prove that $3x = 7y$.

19.

1. Simplify

$$(1) \frac{a}{2} \left(\frac{b}{3} - \frac{c}{4} \right) - \frac{2b}{3} \left(\frac{a}{2} - \frac{c}{6} \right) + \frac{3c}{4} (2a - b).$$

$$(2) \frac{a-1}{5} - \frac{2}{3}(2+a) - \frac{a-3}{10} + a - 1.$$

$$(3) 3x^2y \div (-2xy^2) + (2x)^2 \div (-xy).$$

2. It can be proved that the first x numbers of the series 1, 2, 3, 4, 5, ... add up to $\frac{1}{2}x(x+1)$. Use this result to obtain the values of

$$1 + 2 + 3 + \dots + 9 + 10; \quad 11 + 12 + \dots + 19 + 20;$$

$$100 + 101 + \dots + 199 + 200; \quad 1 + 2 + 3 + \dots + 2n;$$

$$k + 1, k + 2, \dots k + n;$$

where n and k are positive integers.

3. If $x - 2y = 3$ and $2y - z = 4$ and $z - 2w = 5$ and $w + 1 = 0$, find the values of (1) x , (2) $x + y + z$.

4. Solve the equations:

$$(1) 3(1 - x) - 4(x + 8) = 0.$$

$$(2) \cdot 2(x - \cdot 1) = \cdot 3(x - \cdot 2).$$

$$(3) \frac{x+1}{3} - \frac{x-1}{2} = 1 + \frac{2x}{3}.$$

5. The ages of three brothers are 8, 10 and 12 years, and their father is 49 years old. In how many years' time will the sum of their ages be three-quarters that of their father?

20 TEST PAPERS IN ELEMENTARY ALGEBRA

6. A boy counts 2 marks for every sum he gets right and -1 for each that he gets wrong. After doing 25 sums, he finds he has 29 marks. How many did he get right?

7. If a train is x feet long, how many seconds will it take to cross a viaduct, of length y feet, if it is travelling z miles an hour. First do this question for the case when $x=150$, $y=550$, $z=30$.

8. Prove that 1 , -3 and $\frac{1}{2}$ are the solutions of the equation $2x^3 + 3(1+x^2) = 8x$.

20.

1. If $a = \frac{1}{2}$, $b = -2$, $c = 1$, $d = 0$, find the values of

(1) $a^2bc - abc^2 + (abc)^2$. (2) $a(b-c) + d(a-c)$.

2. (1) Subtract the sum of $2a(a+b) - b^2$ and $3a^2 - 2b(a-b)$ from the sum of $3a^2 - 2ab - b^2$ and $2a(3a-b) + 2b^2$.

(2) What must be added to $4a^2 - 6ab - 3b^2$ to obtain $5a^2 + 3ab - 7b^2$?

3. Simplify (1) $\frac{2x}{y} + \frac{3y}{x} - \frac{3xy}{y^2}$.

(2) $\frac{x-5}{6y} + \frac{2x-1}{9y} - 2 - \frac{x-2y}{12y}$.

4. If the length, breadth and height of a rectangular block of stone are x feet, y feet, z feet, and if its weight is w tons, you are given that $w = 2xyz$. Find the height of a block that weighs 150 tons if it is 4 feet long and 3 feet wide.

5. By selling a horse for 60£ a man gains five times as much as he would lose if he sold it for 42£. What did it cost him?

6. Solve the equations :

$$(1) 1.2x - 3.4 = .05x + 1.2.$$

$$(2) \frac{3x-1}{5} - \frac{1+x}{2} = 3 - \frac{x-1}{4}.$$

$$(3) \frac{7}{3x} - 5 = \frac{1}{2} - \frac{5}{x}.$$

7. A room is 25 feet long and 16 feet wide, and the area of the four walls is greater than the area of the floor and ceiling by 143 square feet. Find its height.

8. Find x in terms of a and b if

$$(1) ax = b.$$

$$(2) bx + a = 0.$$

$$(3) ax + b = bx + a.$$

$$(4) a(x-1) + b(x+1) = 0.$$

PAPERS 21-30 include

H.C.F. and L.C.M. of single terms; multiplication and division; simultaneous equations; graphs of simple functions; fractions with monomial denominators.

21.

1. (1) Multiply $4x^3 - 2x^2 - x + 1$ by $3x^2 - 2x - 1$.

(2) Divide $3x^4 - x^3 - 5x^2 + x + 2$ by $x - 1$.

2. If $\frac{x+\tilde{y}}{2} - \frac{x-y}{3} = 1 - \frac{2x+y}{6}$, find the values of x when

$$y = 3, 2, 1, 0, -1, -2.$$

3. A lends $3x - 2y - 3z$ £ to B , $5y - 2x + z$ £ to C and $10x - 2z$ £ to D ; they agree to repay him at the end of a year with 25 per cent. interest; how much will A receive? If D goes bankrupt at the end of the year and is only able to pay

22 TEST PAPERS IN ELEMENTARY ALGEBRA

ten shillings in the pound, how much money does A lose altogether?

4. Solve the equations :

$$(1) \frac{x-1}{.3} = 1 + \frac{x+1}{.2}.$$

$$(2) (x+1)(x-2) + 5 = (1-x)(2-x).$$

$$(3) \frac{x-a}{b} + \frac{a}{2b} = 1 - \frac{x}{2b}.$$

5. Prove that the difference of the cubes of two consecutive integers exceeds three times their product by unity. Verify this for the numbers 5^3 , 6^3 .

6. If $y = 2x - 3$, find the values of y when

$$x = 3, 2, 1, 0, -1, -2, -3,$$

and represent them graphically. From your graph read off the values of y when $x = \frac{3}{2}$, $-\frac{3}{2}$, 4.

7. A bag contains x sixpennies, x shillings and $2x - 3$ florins, and its contents are worth 3£. Find the value of x .

8. A man walks from X to Y at 4 miles an hour, and returns the next day to X at $3\frac{1}{2}$ miles an hour, taking an hour longer for the journey than on the previous day. How far is it from X to Y ?

22.

1. Simplify

$$(1) (x+1)(x+2) - (x-1)(x+3) + (1-x)(x-1).$$

$$(2) (x^4 - x^3 + x^2 + 3x) \div (x^2 + x).$$

Check your answer to each for the case $x = 2$.

2. Find the H.C.F. and L.C.M. of $9a^3xy$, $6a^2x^2y^2$, $15a^4y^3$;

and simplify
$$\frac{a+b}{ax} - \frac{a-b}{bx} - \frac{a^2+b^2}{abx}.$$

3. A man walks out x miles at 4 miles an hour and back again at 3 miles an hour. Find his average pace.

4. Solve the equations :

$$(1) \ 3(x-1)(x-2) = (1-x)(1-3x).$$

$$(2) \ \frac{2x-1}{5} - \frac{1+x}{2} = 1.$$

$$(3) \ x+2y=8, \ 2x+y=7.$$

5. What integral values of x will make each of the expressions $\frac{2x}{3} - 1$ and $13 - \frac{x}{4}$ greater than 8?

6. If a, b, c are three consecutive integers, prove that $ab+bc=2b^2$ and that $a^2-2b^2+c^2$ has the same value whatever a is equal to.

7. If A gives B 12 shillings, A will have left $\frac{1}{3}$ of what A and B together have. If B gives A 6 shillings, A will have twice as much as B . How much has each?

8. In a college made up of scholars and commoners, one-fifth of the number of commoners is greater than the number of scholars by 6, and six times the number of scholars exceeds the number of commoners by 40. Find the number of each.

23.

1. Prove that

$$(1) \ (x+y)(x-y) = x^2 - y^2. \quad (2) \ (x+y)^2 = x^2 + 2xy + y^2.$$

$$(3) \ (x-y)^2 = x^2 - 2xy + y^2.$$

$$(4) \ (x^2 + xy + y^2)(x^2 - xy + y^2) = x^4 + x^2y^2 + y^4;$$

and simplify $(x+y)(x^2 - xy + y^2).$

2. State in words the results mentioned in parts (1), (2), (3), of Question 1.

R.B.A.N. M'S HIGH SCHOOL

Bangalore.

24 TEST PAPERS IN ELEMENTARY ALGEBRA

3. A room is x feet long and y feet wide: a carpet in the middle of the room leaves a margin 2 feet wide all the way round. Find the area of the margin.

4. Solve the equations:

$$(1) (x+1)^2 + x^2 + 1 = (x-1)^2 + x^2 - 1.$$

$$(2) 3\left(\frac{x}{4} - 1\right) - 2\left(\frac{x}{3} + 1\right) = \frac{x}{6} - \frac{2}{3}.$$

$$(3) \frac{1}{2}x + \frac{1}{3}y = 1; \quad 2x + 3y + 1 = 0.$$

5. A force of 600 is made up of three divisions; the second division is half as large again as the first, and the third less by 50 than half the second. How many men are in each division?

6. If $y = \frac{x^2 - 4x}{5}$, find the values of y when

$$x = 6, 5, 4, 3, 2, 1, 0, -1, -2, -3,$$

and represent them graphically. From your figure read off the value of y when $x = \frac{3}{2}$ and the values of x when $y = 2$.

7. Prove that the difference of the cubes of two consecutive even numbers is less by four than three times the sum of their squares.

8. A man saves annually one-fifteenth part of his income. When his income is increased by 150£ a year, he saves one-twentieth part. If this is £2. 10s. more than before, find his original income.

24.

1. If $x = a - 2b$, $y = 2a - b$, prove that

$$x^2 - 2xy + y^2 = a^2 + 2ab + b^2 \quad \text{and that} \quad \frac{xy + ab}{a - b} = \frac{2}{3}(x + y).$$

2. C is the middle point of a line AB and D is any point on AB . Prove that $AD^2 + DB^2 = 2AC^2 + 2CD^2$. [Let AC

equal x inches and $CD = y$ inches, and express each side in terms x and y .]

3. Simplify ,

$$(1) 2x(x - y) - 3x(x - [y - x] - 2y) + (x - y)^2.$$

$$(2) \frac{1}{3}(3x - 7) - \frac{1}{5}(5x - 8) + 4.$$

4. A man buys $x - y$ horses for $x + y$ pounds each, and $x - 2y$ horses for $3y$ pounds each ; he sells them all for $2x - y$ pounds each. How much profit does he make ?

5. Solve the equations :

$$(1) \frac{1}{x} + \frac{2}{5x} = .1.$$

$$(2) 4x - \frac{1}{5}(x + 4) = 41.$$

$$(3) 4x - 2y = 4y - 5x = x + y - 3.$$

6. One angle of a triangle is four times another angle, and the remaining angle is equal to half the sum of the other two. Find the three angles.

7. a, b, c, d are four consecutive integers such that cd exceeds ab by 54. Find the value of a .

8. Draw a graph to show the values of the expression $2x - 3$ as x varies from -1 to $+4$, and read off from the figure the value of x when the expression is equal to 4.

25.

1. Divide the product of $x^2 - 3x + 2$ and $x^2 - 2x - 3$ by $x^2 - 1$. Check your answer for the case $x = 4$.

2. If $2x - y = 1$ and $3x + 2y = 3$, prove that $x + 2y = 1\frac{4}{7}$.

3. What is the next whole number above (1) 196, (2) x^2 that is a perfect square, x being an integer ?

If a, b, c are three consecutive square whole numbers, prove that $a^2 - 2b^2 + c^2 = 2$.

26 TEST PAPERS IN ELEMENTARY ALGEBRA

4. Simplify (1) $x(x+1)^2 - x^2(x+1) - (x-1)(x+1)$.
 (2) $(1+x)[1-x(1+x)+x] + (1-x)(1+x^2)$.
 (3) $\frac{2x-y}{3xy} - \frac{x-2y}{x^2} + \frac{3x-y}{2y^2}$.

5. A man spends annually five times as much money as he saves. If he spent 40£ less each year he would save one-quarter of his income. Find his income.

6. Solve graphically the equations $y = 2x - 5$, $x = 2y + 1$, and check your result algebraically.

7. Solve the equations :

- (1) $x(x-1)(x-2) = (x-5)(x+1)^2$.
 (2) $\frac{1}{x} - \frac{1}{y} = 2$, $\frac{1}{x} + \frac{1}{y} = 8$.

8. A and B have to pay a bill of 1£. 2s. ; A can pay it if B gives him one-quarter of what he has, and B can pay it if A gives him one-third of what he has. How much has each ?

26.

1. Find the square of $2x - 3y$.

Of what is $25x^2 - 30xy + 9y^2$ the square ?

How much must be added to $9x^2 + 4y^2$ to make it a perfect square, and what is its square root ?

2. From the sum of $\frac{1}{2}x - \frac{1}{3}y + \frac{3}{4}z$ and $\frac{2}{3}x - \frac{1}{2}z$ subtract the sum of $y - \frac{1}{3}x$ and $2x - \frac{1}{4}z$.

Check your answer by putting $x = 6$, $y = 3$, $z = 4$.

3. If $y = \frac{2x+5}{9x-2}$, find x in terms of y .

4. (1) If $\frac{x}{3} = \frac{y}{5} = \frac{z}{10}$, find the value of $\frac{x+y}{4x-z}$.

(2) If $x + \frac{1}{x} = 3$, find the values of $x^2 + \frac{1}{x^2}$ and $x^3 + \frac{1}{x^3}$.

5. Solve the equations :

$$(1) \frac{2}{3x+3} = 1 - \frac{3}{2x+2}.$$

$$(2) 3x - 5y = 4 ; 2(3 - x) = 3 + x.$$

6. If the sum of two fractions is unity, prove that the sum of the first and the square of the second is equal to the sum of the second and the square of the first.

7. Draw the graph of the function $(x-1)(x-4)$ for the range $x = -1$ to $x = 6$. Read off from your figure the least value of the function and also the values of x for which the function is equal to 2.

8. Winchester is 66 miles from London. A train leaves Winchester for London at 9.10 a.m., and travels 40 miles an hour ; at 9.25 a.m. another train, travelling 44 miles an hour, leaves London for Winchester. How far from London will they meet ?

27.

1. (1) Divide $x^{12} + x^6 - 2$ by $x^4 + x^2 + 1$.

(2) Simplify

$$(a+b+c)(ab+bc+ca) - (a+b)(b+c)(c+a).$$

2. A stone, when dropped, falls $16t^2$ feet in t seconds. How far will it fall in 3 seconds, and how far in the fifth second ?

One second after the first stone is dropped, a second stone is let fall. How long after the second stone has started will it be 80 feet behind the first ?

3. If $x = 3$, $y = -2$ are the answers to the equations $x + ay = 5$; $bx + y = 7$, find the values of a and b .

28 TEST PAPERS IN ELEMENTARY ALGEBRA

4. Solve the equations :

$$(1) \cdot 01(2x - 4) = \frac{3}{4}(x - 2).$$

$$(2) \frac{3x - 1}{7} - \frac{2x + 3}{4} = \frac{x - 20}{12}.$$

5. O, A, B, C, D, E are six points on a straight line such that $AB = BC = CD = DE$. Prove that

$$OA^2 + OB^2 + OD^2 + OE^2 = 4OC^2 + AD^2 + DE^2.$$

6. x and y are two whole numbers, such that when x is divided by y the quotient is 4 and the remainder 5, and when $10y$ is divided by x the quotient is 2 and the remainder 4. Prove that $2x - 9y$ is a factor of $x - 3y$.

7. The function $a(x^2 - 1) + bx$ is equal to 3 when $x = 1$ and to 12 when $x = 2$. Calculate a and b , and draw the graph of the function from $x = -3$ to $x = 2$; read off from the graph the values of x for which the function is equal to 7.

8. Find the fraction which reduces to $\frac{1}{2}$ when 1 is added to the denominator and reduces to $\frac{2}{3}$ when 2 is added to the numerator.

28.

1. (1) If $2x - 3y = 4$ and $3x + y = 1$, find the value of $\frac{x}{y}$.

(2) Divide

$$(x + a)(x - 2a) + (x + 3a)(x - a) - (x + a)^2 \text{ by } x - 3a.$$

2. Simplify (1) $\frac{x+1}{2x} + \frac{3x+1}{5} - \frac{1+2x}{3x}$.

$$(2) \left(x + \frac{1}{x} - 1\right) \left(\frac{1}{x} + \frac{1}{x^2}\right).$$

3. B is the mid point of the line AD and C is the mid point of BD . Prove that

$$AD^2 - AC^2 + CD^2 = AC \cdot BD + AB \cdot BC.$$

4. If x half-crowns and y shillings are worth twice as much as y half-crowns and x shillings, and are also worth 12 shillings more than $(x + y)$ shillings, find the values of x and y .

5. Solve the equations :

$$(1) \cdot 01(x - 5) = \cdot 24(3 - x).$$

$$(2) \frac{x-1}{7} - \frac{2x+1}{5} = \frac{x-10}{12} + \frac{1-x}{3}.$$

6. Draw the graph of the function $\frac{1}{5}(x-2)(x+2)$ between $x=4$ and $x=-4$, and read off from the figure the values of x for which the function is equal to 2.

7. A man is engaged to work for 40 days ; he is paid four shillings a day when he works and is fined one shilling a day when absent. At the end he receives 6£. How many days did he work ?

8. If x men take y days to finish a certain job and if a women do as much work as b men, how long would z women take ?

29.

1. (1) Divide $x^6 - 64x^4 - 4x^2 + 256$ by $x^2 - 2$.

(2) Find the continued product of

$$x^2 + xy + y^2 ; \quad x^2 - xy + y^2 ; \quad x^2 - y^2.$$

2. Solve the equations :

$$(1) \frac{x+1}{3} - \frac{x-4}{7} = \frac{x+4}{5}.$$

$$(2) 2x - y - 3 = 3y - x + 4 = 5x - 8y - 4.$$

3. If the straight line AC is bisected at B and is produced to any point D , prove that $BD^2 + CD^2 = AB^2 + 2CD \cdot BD$.

4. A is 35 years old, B is 7 years old. In how many years' time will A be twice as old as B ?

30 TEST PAPERS IN ELEMENTARY ALGEBRA

5. (1) If $5x = 2$, find the value of $\frac{1 + 2x + \sqrt{1 - 4x^2}}{1 + 2x - \sqrt{1 - 4x^2}}$.

(2) If $3x - y = 9$ and $x - 2y = 8$, find the value of $\frac{x + y}{x - y} \cdot \left(\frac{1}{x} - \frac{1}{y} \right)$.

6. Simplify

(1) $(x - 2)(x + 3)(x + 5) - (x + 1)^2(x + 4)$.

(2) $\frac{3a - b}{ab} - \frac{2b - c}{bc} + \frac{c - 2a}{ac}$.

7. On a railway the telegraph poles are placed x yards apart; if a train passes y poles in a minute, find the speed of the train in miles per hour.

8. A messenger starts on an errand at $3\frac{1}{2}$ miles an hour; 40 minutes later a boy bicycles after him at 14 miles an hour. Where will he overtake the messenger?

30.

1. (1) Multiply $x^2 - 3 - \frac{2}{x}$ by $2x + 1 + \frac{3}{x}$.

(2) Divide $1 - 6x^5 + 5x^6$ by $1 - 2x + x^2$.

2. $ABCD$ is a straight line. If AB and CD are of equal length, prove that $AD^2 - BC^2 = 4AC \cdot CD$.

3. The first man home in a steeplechase gets 100 marks, the next 99, the next 98, and so on. How many marks will (1) the 10th man, (2) the 35th man, (3) the x th man get?

4. (1) Find the H.C.F. of $40x^2yz^3$, $60x^2y^2z^2$, $32xy^3z^4$.

(2) Simplify $\frac{3x - 1}{7} - \frac{2x + 4}{6} + \frac{8 - 9x}{21} + x$.

5. Solve the equations :

$$(1) \quad 3(2x - 5) - 5(4 - x) = \frac{1}{2}x + 7.$$

$$(2) \quad \frac{x}{2} - \frac{y}{3} = \frac{2x}{5} - \frac{y}{4} = 3x - 2y - 5.$$

6. Find a number between 700 and 800, such that its excess over the number formed by reversing the digits is $\frac{5}{8}$ of the original number, the sum of the digits being 18.

7. How many minutes is it before six o'clock, if an hour ago it was twice as many minutes past four?

8. If a, b, c, d are four consecutive odd integers in ascending order of magnitude, prove that $bc - ad = 8$ and $ac - bd = b^2 - c^2$.

PAPERS 31-40 include

Easy factors; graphical solution of simultaneous equations; quadratic graphs.

31.

1. (1) If $x = 3a - 5$ and $y = 7 - 2a$, what value of a will make x and y equal?

(2) If $4x + 8y = 5$ and $3x + 7y = 4$, find the value of $\frac{x}{y}$.

2. Simplify

$$(1) \quad \frac{2x}{y} + \frac{3y}{x} - 4 - \frac{x^2 + y^2}{xy}.$$

$$(2) \quad \frac{2a + 2b - c}{ab} + \frac{3a + b - 2c}{ac} - \frac{a - 5b}{bc}.$$

3. Find the factors of

$$(1) \quad 3x^3yz - 2xy^3 + 6x^2y^2.$$

$$(2) \quad x^2 - 11x + 28.$$

$$(3) \quad 4x^2 - 9b^2.$$

32 TEST PAPERS IN ELEMENTARY ALGEBRA

4. If you have $2a + 3b$ coins, of which $a - 4b$ are half-crowns and the rest shillings, what is the value of the coins in pounds?

5. Solve the equations :

$$(1) \frac{2x-1}{5} - \frac{3x+1}{2} = \frac{2}{5}.$$

$$(2) (x+1)(2-x) + (3+x)(2x+1) = (x+2)(x-3).$$

6. A man walks up a hill at 2 miles an hour, and then back home at 5 miles an hour; he takes 4 hours 12 minutes altogether. How far did he go?

7. Prove that the difference of the squares of two consecutive integers is equal to the sum of the integers.

8. Prove that the equation

$$(2x+1)(x+3) - (x-1)(3x+2) + (x+2)^2 = 3(4x+3)$$

is satisfied by every value of x .

32.

1. (1) To $(a+2b)x + (2a-c)y + (c-b)z$
add $(x-2y)a + (z-2x)b + (2y-z)c$.

(2) Add together $\frac{2}{3}(a-1) + \frac{1}{4}(1+b)$; $\frac{1}{2}(3a-5b)$;
 $a+b-1$.

2. (1) Divide $x^4 - 7x^2y^2 + y^4$ by $x^2 + 3xy + y^2$.

(2) What are the coefficients of x^6 and x^3 in the expression $(2x^4 - 3x^3 - x^2 + 1)(3x^3 + x^2 - x - 2)$.

3. Find the factors of

$$(1) ax - 2by - 2bx + ay.$$

$$(2) x^2 - 12x - 45.$$

$$(3) 2x^2 - 18y^2.$$

4. A clock loses x seconds a day; it is set right at noon. What time will it show when the real time is y minutes past z o'clock the same day?

5. A man bicycles from A to B at 12 miles an hour: he takes 20 minutes longer over one-third of the distance than over one-fifth of it. How far is it from A to B ?

6. If $y = ax(x+1) + b$ and if $y = 3$ when $x = -2$ and if $y = 2$ when $x = 2$, find the value of y when $x = 3$.

7. Solve graphically the equations $y = 3x - 2$, $x = 1 - 2y$, and then solve them algebraically.

8. A can check the answers to 30 questions in 12 minutes, and B can check the answers to 25 questions in 15 minutes. If they both set to work together, how long will the checking take for 850 questions?

33.

1. (1) Divide $x^5 - 26x^3 + 22x^2 + 24x + 4$ by $x^2 - 4x - 2$.

(2) Multiply $a^2 - (b-c)^2$ by $a^2 + (b+c)^2$.

2. Find the factors of

(1) $x^2 - xy + 2x - 2y$.

(2) $x^3 + x^2 + x + 1$.

(3) $x^2 - 15x + 54$.

(4) $74^2 - 22^2$.

3. Simplify

(1) $\frac{1}{3x}(2x-5y) + \frac{2}{5x}(3x+4y) - \frac{9}{10}$;

(2) $s^2 + (s-a)^2 + (s-b)^2 + (s-c)^2$,

if $2s = a + b + c$.

4. A bookshelf is long enough to hold x books of one size or y books of another size. If a books of the first kind and b books of the second just fill it, prove that $\frac{a}{x} + \frac{b}{y} = 1$.

5. Solve the equations:

(1) $\frac{1}{2x} - \frac{1}{3x} = \frac{1}{5}$.

(2) $\frac{3}{x} - \frac{1}{y} = 1$; $\frac{5}{x} + \frac{2}{y} = 20$.

(3) $\left(\frac{x-1}{x+1}\right)^2 = \frac{x-4}{x}$.

34 TEST PAPERS IN ELEMENTARY ALGEBRA

6. Two railway routes between two towns differ by 1 mile. A train travelling 48 miles an hour by the longer route takes 4 minutes less than a train travelling 45 miles an hour by the shorter route. Find the length of each route.

7. B is 5 miles due east of A ; A walks 4 miles an hour due south, and B walks 3 miles an hour due west. Find their distance apart after x hours, and prove that their distances apart after $36 - y$ minutes and $36 + y$ minutes are equal.

8. Draw the graph of the function $(x+1)(x-2)$ between $x=5$ and $x=-4$, and obtain from it the values of x , for which the function is equal to 8.

34.

1. Simplify $(p+q)(q+x) - (x+y)(p+y) - (p+x)(q-y)$.

2. Solve the equations:

$$(1) \frac{3x-1}{5} = \frac{2-4x}{9}. \quad (2) \cdot 07(x - \cdot 3) = \cdot 2(\cdot 12 - x).$$

$$(3) \frac{x-1}{3} + \frac{y+1}{2} = 1, \quad \frac{2x+1}{5} - \frac{3y+1}{4} = 5.$$

3. Find the factors of

$$(1) 5x^2 - 5ax - 3x + 3a. \quad (2) (x-4)(x-5) - 2.$$

$$(3) 3x^2 - 27.$$

4. Two trains start at the same time to run from A to B and from B to A respectively; they meet after $2\frac{1}{2}$ hours, and the first train arrives at B 1 hour later. Find how long the second train takes over the journey.

5. Solve graphically the equations:

$$5x - 2y = 7; \quad 2x + 5y = 6.$$

Check your result algebraically.

6. Find a quadratic function of x which has the values 0, -1 , 2, when x equals 1, 0, -1 respectively; and prove that the function is zero when x equals $-\frac{1}{2}$.

7. One man leaves A at noon, walking 4 miles an hour, and a second man leaves B at 1.45 p.m., cycling 12 miles an hour. The distance from A to B is 31 miles. Where will they meet?

8. If $s = ut + \frac{1}{2}ft^2$, find f when $s = 150$, $u = 10$, $t = 5$; also find u , if $s = 100$, $t = 8$, $f = 1\frac{1}{2}$.

35.

1. (1) Simplify

$$(a-b)(x-y) + (a+b)(x+y) - (a-x)(b+y) - (a+y)(b+x).$$

(2) Prove that $\frac{x^3 - y^3}{x - y} - \frac{x^3 + y^3}{x + y} = 2xy$.

2. (1) Multiply $\frac{2x-1}{3} - \frac{1-x}{2} + \frac{3}{4} - \frac{1}{6}x$ by 12.

(2) Simplify $3 - \frac{x-y}{2x} + \frac{2x+y}{3y} - \frac{4x^2-y^2}{6xy}$.

3. The sum of the squares of the first n positive integers is $\frac{1}{6}n(n+1)(2n+1)$. Use this formula to write down the values of

(1) $1^2 + 2^2 + 3^2 + \dots + 9^2 + 10^2$.

(2) $20^2 + 21^2 + 22^2 + \dots + 29^2 + 30^2$.

(3) $a^2 + (a+1)^2 + (a+2)^2 + \dots + (2a)^2$.

4. Put into factors:

(1) $b(6a-3b) + bc - 2ac$.

(2) $x(x+1) + (x+4)(x-3)$.

(3) $(x+1)^2 - (x+1)(2x-3)$.

(4) $(x+y)^2 - 9y^2$.

5. $x\text{£}$ is distributed between A , B and C . A receives twice as much as B , and C receives 10£ less than A . Find, in terms of x , the amount C receives.

6. Solve the equations :

$$(1) \frac{1}{7}(2x-1)+2=\frac{1}{15}(7x-2). \quad (2) \frac{x}{\frac{2}{3}}-\frac{x-1}{\frac{2}{5}}=\frac{3}{4}.$$

$$(3) \frac{2x-y}{3}=\frac{x-3y}{2}, \quad \frac{x}{3}=1-y.$$

7. One-fifth of the length of a post is in the mud, one-half of its length is in the water, and the part above the water is 4 feet longer than the part in the mud. Find the length of the post.

8. Some boys were given a certain number of oranges to divide between themselves. If they took ten each, there were three left over, but there were four too few for them to have eleven each. Find the number of boys.

36.

1. If $y=2x-3$ and $x(x-y)=1$, prove that $x^2-3x+1=0$ and that $y^2=5$.

2. Simplify (1) $(x-1)(x+1)^2-x^2(x+2)$.

$$(2) \frac{2x-y}{\frac{3}{4}}-\frac{3x+y}{\frac{1}{2}}-\frac{y-3x}{\frac{3}{5}}.$$

3. (1) Divide $x^4+x^2y^2+y^4$ by x^2+xy+y^2 .

(2) Multiply

$$3a(a-b)-2b(2b-a) \text{ by } 3(a-b)-2(2b-a).$$

4. $ACBD$ is an oval track x yards in perimeter ; C divides one of the arcs AB in the ratio $2:3$, and D divides the other arc AB in the ratio $3:7$. If the two arcs AB are of equal length, find the length of the arc CBD , and the difference between AC and BD .

5. Find the factors of

$$(1) x(x+4y)-3y(3x-2y).$$

$$(2) 5x^6y^4-15x^2y.$$

$$(3) 6x^2+x-1.$$

$$(4) x^2-(x-3y)^2.$$

6. If a, b, c are three consecutive integers, a being the least, prove that

$$(1) 2b = a + c; \quad (2) a^2 + 2c = b^2 + 3; \quad (3) \frac{1}{2}(ab + bc)$$

is a perfect square.

7. Solve the equations :

$$(1) \cdot 3(x - \cdot 1) + \cdot 02(x - 4) = \cdot 05.$$

$$(2) \frac{x - 2y}{x} = \frac{3y}{2x}; \quad \frac{3}{x} - \frac{2}{y} = \frac{1}{xy}.$$

8. A train leaves P for Q at noon, travelling 32 miles an hour, and, when it has gone ten miles more than half the distance, meets a train which left Q at one o'clock, travelling 40 miles an hour. Find the time when the second train will arrive at P .

37.

1. (1) Simplify $x^2 - 1 + \frac{1}{4}x(x + 1)(x - 3) + \frac{1}{4}x(x - 1)^2 + \frac{1}{2}x$.

(2) Divide $x^5 + 9x^3 + 81x$ by $x^2 - 3x + 9$.

2. A has x shillings and B has y shillings. How much must A receive from B in order to have twice as much money as B ?

3. Simplify (1) $\frac{2}{3}(3x - 1) - \frac{1}{4}(1 - 2x) - 1 + \frac{1}{2}x$.

$$(2) \left(\frac{1}{x} + \frac{1}{y}\right)(x + y) - \left(2 + \frac{x}{y}\right).$$

4. (1) Find an integral value of x which makes $7x - 1$ greater than $2x + 9$ and makes $8x - 3$ less than $5x + 9$.

(2) Find the least integral value of y for which the expressions $x + 2y - 1$, $2x - y + 4$, $7y - 5x - 2$ are in descending order of magnitude; and determine the limits within which x must lie for this value of y .

38 TEST PAPERS IN ELEMENTARY ALGEBRA

5. Find the factors of

(1) $3x^2 - 6x + 3$.

(2) $x^2y^2 - 4x^3y^3 + 5x^4y^4$.

(3) $(a + 2b - 3c)^2 - (2a - b + 5c)^2$.

(4) $6(x^2 - 1) - 5x$.

6. Solve the equations:

(1) $x(x + 2)(x + 5) = (x + 1)(x + 3)^2$.

(2) $\frac{1}{5}(x - 3) + \frac{1}{3}(y - 7) = x - y = \frac{1}{3}(5x + y)$.

7. A housemaster spends four guineas on golf balls at 2s. 6d. and 2s. for his house, which contains 38 boys. If each receives one, how many get the more expensive kind?

8. If a cricketer scores 72, his average will rise by 3; but if he only scores 6, it will fall by 3. What is his present average?

38.

1. (1) If $x = 3$, $y = -1$, $z = -2$, prove that

$$x^3 + y^3 + z^3 = 3xyz.$$

(2) If $x = y - z$, prove that $x^3 + z^3 = y(y^2 - 3xz)$.

2. If $x = 3$ is one answer to the equation

$$\frac{x-1}{x+2} + \frac{2x-a}{2x-1} = \frac{x+a}{x-2},$$

find the value of a .

3. If x miles in y hours is the same pace as p feet in q seconds, find in its simplest form an equation connecting p , q , x , y .

4. Find the factors of

(1) $x^2 - 11x + 24$.

(2) $x^2 - 11x - 42$.

(3) $2x^2 - 18y^2$.

(4) $x^2 + x + \frac{1}{4}$.

(5) $x^2 + (a - b)x - ab$.

5. Find a quadratic function of x which vanishes when $x=3$ and takes the values -3 , -2 , when x equals 1, 2 respectively.

6. For what value of x is $2x^2 + 3x - 1$ a factor of

$$2x^4 + x^3 + 8x - 5?$$

7. In a certain congregation there were three times as many women as men. Three men went out before the sermon, leaving four times as many women as men. How many men stayed for the sermon?

8. In exchange for x sovereigns a man may be paid either y half-crowns and z florins, or $\frac{y}{2} + 10$ half-crowns and $2z - 25$ florins, or $\frac{6y}{5} - 8$ half-crowns and $\frac{4}{5}z$ florins. Find x .

39.

1. Assuming that the sum of the cubes of the first n integers is equal to $\left[\frac{n(n+1)}{2} \right]^2$, find the values of

$$(1) 1^3 + 2^3 + 3^3 + \dots + 8^3 + 9^3;$$

$$(2) 10^3 + 11^3 + 12^3 + \dots + 20^3;$$

$$(3) a^3 + (a+1)^3 + \dots + (a+7)^3 + (a+8)^3,$$

where a is a positive integer.

2. Prove that

$$(1) x^2 - x - y^2 - y = (x+y)(x-y-1).$$

$$(2) (x-2y)(3x+y) - (3x-y)(x+2y) + (x+2y)^2 + y^2 \\ = (x-2y)(x-3y).$$

3. B is x years older than A ; two years ago he was four times as old as A , and in five years' time he will be three times as old as A . Find x .

40 TEST PAPERS IN ELEMENTARY ALGEBRA

4. Find the factors of

$$(1) x^3 + x^2 + x + 1.$$

$$(2) x^2y^2 - 10xyz - 24z^2.$$

$$(3) (ab - cd)^2 + (ac + bd)^2.$$

$$(4) (ab + cd)^2 - (ac + bd)^2.$$

5. A man has an average of x runs for y innings. How much must he make in his next innings, assuming he gets out, so as to increase his average by one run?

6. Solve the equations:

$$(1) 3(x - 1) - 2(3 - 5x) = \frac{1}{2}x + 4.$$

$$(2) 3 - \frac{1}{x} = \frac{10x + y}{x} = 1\frac{1}{2}.$$

7. Four pounds of coffee cost the same as three pounds of tea. If the coffee cost threepence a pound less and the tea twopence a pound more, five pounds of coffee would cost the same as three pounds of tea. Find the price per pound of each.

8. A cyclist sets out from X at 16 miles an hour, but is delayed for 20 minutes by a puncture, after which he proceeds at 12 miles an hour. A motorist who left X one hour after the cyclist, and averages 20 miles an hour, passes him $32\frac{1}{2}$ miles from X . Where did the puncture take place?

40.

1. A body travels $5t + 2t^3$ feet in t seconds. Find the distance it goes in (1) 3 seconds, (2) the fifth second.

If it travels x, y, z feet in $a, 2a, 3a$ seconds respectively, prove that $5x + z = 4y$.

2. Multiply $x^2 + 2x - 3$ by $x^2 - 3x - 10$ and divide the result by $x^2 - 6x + 5$. Find the value of x for which $x^2 - 6x + 5$ is the square root of the product.

3. Simplify

$$(1) \frac{2x-3y}{\frac{3}{4}x} - \frac{3x+5y}{\frac{2}{5}y} + \frac{3x^2+24y^2}{6xy}.$$

$$(2) \frac{2}{3}a^2bc \times \frac{15ab}{3c^2} \div \frac{14ac}{9b^2} \times \left(\frac{2}{3}a\right)^2.$$

4. Find the factors of

$$(1) a^2 - ab + bc - ac.$$

$$(2) (x-y)^2 - 2x(x-y).$$

$$(3) 12x^2 + 5xy - 72y^2.$$

$$(4) x^2 - \frac{1}{x^2}.$$

5. When a man is walking x miles an hour he takes y steps a minute, but if his pace is p miles an hour he takes q steps a minute. Find in inches the difference between the lengths of his stride in the two cases.

6. Solve the equations :

$$(1) \frac{x-3}{x-5} = \frac{2+x}{3+x}.$$

$$(2) \frac{x-y}{x+y} = \frac{3}{4} = \frac{x-1}{y+2}.$$

7. Find a fraction which reduces to $\frac{3}{4}$ when 1 is added to the numerator, and to $\frac{1}{2}$ when 10 is added to the denominator.

8. There are two roads from A to B . Two men set out at the same time from A for B by different routes and return the opposite way they went ; they travel 10 and 12 miles an hour respectively. If they pass each other ten miles from B , and if one man arrives home 55 minutes before the other, find the lengths of the two roads. [There are two cases.]

PAPERS 41-50 include

Fractions; harder factors; H.C.F. and L.C.M. by factors; simultaneous equations in three unknowns.

41.

1. (1) Prove that $\left(1 + \frac{a}{b}\right) \div \left(1 + \frac{b}{a}\right)$ is the cube root of
- $$\left(1 + \frac{a^3}{b^3}\right) \div \left(1 + \frac{b^3}{a^3}\right).$$

(2) Find the value of $2x^2 - 3xy - 2y^2 - x - 3y - 1$ when $2y + 1$ is substituted for x .

2. If $s = \frac{u+v}{2} \cdot t$, find v when $s = 50$, $u = 8$, $t = 4$, and find t when $s = 210$, $u = 10$, $v = 18$.

3. The lowest scorer in a shooting eight made x points. If he had made y points, the average of the eight would have been z points. Find the actual total made by the eight.

4. Find the factors of

(1) $8x^2 - 2x - 15$.

(2) $x^2 + y^2 - z^2 + 2xy$.

(3) $6a^2 - 2bc + a(4c - 3b)$.

(4) $6 - x - x^2$.

5. Simplify

(1) $\left(\frac{2}{x+1} - \frac{1}{x+2}\right) \div \left(\frac{2}{x+3} - \frac{1}{x+2}\right)$.

(2) $\frac{3}{2x+6} - \frac{5}{3x+9} + \frac{1}{5x+15}$.

(3) $\frac{4x}{x^2-9} + \frac{2}{3-x} - \frac{3}{3+x}$.

6. A is twice the age of B : in 14 years' time their united ages will be five times B 's present age. How long ago was A three times as old as B ?

7. A, B, C, D are any four points in order in a straight line. Prove that $AC \cdot BD = AB \cdot CD + BC \cdot AD$.

8. The value of twenty-three coins, part of which are florins and the rest half-crowns, is $2\frac{1}{2}$ guineas. How many half-crowns are there?

42.

1. What value of x , other than zero, will make $x^4 - x^3 - 5x^2 - 7x - 1$ exactly divisible by $x^2 - 3x - 1$?

2. Prove that

$$\begin{aligned}(x+y)^3 + (y+z)^3 + (z+x)^3 - 3(x+y)(y+z)(z+x) \\ = 2(x+y+z)(x^2+y^2+z^2 - yz - zx - xy) \\ = 2(x^3+y^3+z^3 - 3xyz).\end{aligned}$$

3. Find the factors of

$$(1) x^2 + 5xy + 6y^2 + x + 2y. \quad (2) x^4 - 13x^2 + 36.$$

$$(3) x^2 + \frac{1}{x^2} + 2. \quad (4) 5x^2 + 8xy - 4y^2.$$

4. Solve the equations:

$$(1) \frac{x+3}{x+2} + \frac{x+4}{x+3} = \frac{2x+10}{x+4}.$$

$$(2) x - 2y = 5; \quad 2y + 3 = z; \quad x + z = 4.$$

5. If x miles an hour is the same speed as 1 yard in t seconds, find the numerical value of xt .

6. A man buys a pony and trap and harness for 48£; the pony costs five times as much as the harness, and the trap cost one-third of the price of the pony and harness together. Find the cost of each.

44 TEST PAPERS IN ELEMENTARY ALGEBRA

7. If a, b, c are three consecutive integers, prove that $3b^2 = ab + bc + ca + 1$ and that $a^2 + c^2$ is always greater than $2b^2$.

8. A man has to transport 16 tons of goods in carts and waggons: he finds he will require either 8 waggons and 12 carts, or 6 waggons and 15 carts. How much does each hold?

43.

1. If $v = u + ft$ and $s = ut + \frac{1}{2}ft^2$, prove that

$$s = \frac{u+v}{2} \cdot t \text{ and that } v^2 = u^2 + 2fs.$$

2. If $x = \frac{a+b}{a-b}$, find in terms of a, b the values of

$$\frac{1+x}{1-x} \text{ and } \frac{ax-b}{bx+a}.$$

3. (1) Find the L.C.M. of $x^2 - x$, $x^2 - 5x + 4$, $x^3 - 16x$.

(2) Simplify $\frac{x^2 + y^2}{x^2 - y^2} - \frac{x + y}{x - y}$.

4. $ACBD$ is a straight line, and $AC = \frac{x}{11}$, $AB = \frac{x}{7}$, $AD = \frac{x}{3}$. If O is the mid point of AB , prove that $OB^2 = OC \cdot OD$.

5. Solve the equations:

$$(1) \frac{3x}{x+1} + \frac{2x}{1-x} = 1.$$

$$(2) 2x + y = 3z + 4y - x = 2y + z + 2, \quad y - z = 6.$$

6. Find the factors of

$$(1) (1+x)^2(1+y^2) - (1+x^2)(1+y)^2.$$

$$(2) x^4 - 26x^2 + 25.$$

$$(3) x^3 - 27.$$

7. A leaves X at 9 a.m. and arrives at Y at 1.20 p.m.; B leaves Y at 10.30 a.m. and arrives at X at 2.30 p.m.; each of them walks at a uniform pace. At what time do they pass each other?

8. A number of two digits is $\frac{4}{7}$ of the number formed by reversing the digits. What is the number? Give all the possible answers.

44.

1. If $a = x^2 - yz$, $b = y^2 - zx$, $c = z^2 - xy$, prove that
 $ay + bz + cx = 0$ and that $\frac{bc - a^2}{x} = \frac{ca - b^2}{y} = \frac{ab - c^2}{z}$.

2. Simplify

$$(1) \frac{2x}{3x-6} - \frac{3x}{5x-10} + \frac{x}{2-x}.$$

$$(2) \frac{x^2 - 4x + 3}{x^2 - 7x + 10} \times \frac{x^2 - 9x + 14}{x^2 - 3x + 2} \div \frac{x^2 - 10x + 21}{x^2 - 4}.$$

3. If the interest on $x\text{£}$ for y years is $z\text{£}$, find the interest on c shillings for b months. Give the answer in pence.

4. Find the square root of

$$(x^2 + 3x - 10)(x^2 + 2x - 15)(x^2 - 5x + 6),$$

and the L.C.M. of $x^2 + 3x$, $x^2 - 9$, $x^2 + 4x - 21$, $x^4 - 49x^2$.

5. A sum of money consists of half-crowns and florins. If the florins were half-crowns and the half-crowns florins, the value would be one shilling less, and if the half-crowns were sixpennies and the florins were crowns, the value would be four shillings more. What is the sum of money?

6. Solve the equations:

$$(1) \frac{x^2 - 1}{3} + \frac{2x^2 + 7}{5} = \frac{x^2 + 4}{2}.$$

$$(2) \frac{3x - 7}{4} = \frac{2y + 1}{3} = 5.$$

7. Prove that $x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$ is unaltered in value if x , y , z are increased by equal amounts.

46 TEST PAPERS IN ELEMENTARY ALGEBRA

8. I borrow equal sums for a year from A , B and C . A asks no interest, B asks 5 per cent., C asks 3 per cent. per annum. At the year's end I have to pay 77£ in all. What did I borrow from each?

45.

1. If $a + b = 3$ and $ab = -2$, find the values of

$$(1) \frac{1}{a} + \frac{1}{b} \qquad (2) a^2 + 2ab + b^2.$$

$$(3) a^2 + b^2. \qquad (4) a^3 + b^3.$$

[Do not try to find the values of a or b separately.]

2. A certain expression is divided by $x^2 - 3x - 1$; the remainder is $2x - 1$, and the quotient is $2x + 1$. Find the remainder when this expression is divided by $x - 1$.

3. A boy is 4 years old and his father's age is 40. After how many years will the father be three times as old as his son?

4. Find the factors of

$$(1) c(x + y) - xy - c^2. \qquad (2) (x - 1)^2 - (x - 1)(2x + 1).$$

$$(3) 3x^2 - 11x + 10. \qquad (4) ax^2z - 16ay^2z.$$

5. Simplify (1) $\left(\frac{2}{x-1} - \frac{1}{x+1}\right) \div \left(\frac{3}{x-1} - \frac{2}{x}\right).$

$$(2) \left(x + \frac{6}{x} - 5\right) \div \left(x - \frac{6}{x} + 1\right).$$

$$(3) \frac{2y}{y^2 - x^2} + \frac{1}{x - y}.$$

6. A has x £ and B has y £; if A gives B 3£, B will have twice as much as A . Find a relation between x and y .

If A and B had between them 30£, find how much B had more than A at the beginning.

7. Solve the equations :

$$(1) \frac{x+2y}{2x-y} = \frac{3x-2}{y-7} = 8.$$

$$(2) 2x - y = 5; \quad 2y - z + 3 = 0; \quad 2z - x = 0.$$

8. A , B , C earn between them 9 shillings a day ; B earns sixpence more in 5 days than C earns in 8 days ; and A earns in 7 days 3 shillings more than B and C together earn in 8 days. How much does each earn daily ?

46.

1. A bicycle wheel makes x revolutions a minute ; how many seconds does it take to turn through y degrees ? If the bicycle travels a miles in b hours, what is the length in feet of the circumference of the wheel ?

2. (1) If $x = 2a - b$ and $y = a + 2b$, find in terms of a , b the value of $\left(\frac{2}{x+y} - \frac{1}{x-y}\right) \div \left(\frac{3}{x-y} - \frac{2}{x+y}\right)$.

(2) If $x = \frac{a-b}{a+b}$ and $y = \frac{2a-b}{a+2b}$, find in terms of a , b the value of $\frac{x-y}{x+y}$.

3. Find two consecutive numbers such that one-seventh of the smaller is less by 5 than one-fifth of the larger.

4. (1) Find the L.C.M. of $15x^2 - 60$; $3x^2 - 3x - 6$; $5x^2 + 5x$.

(2) What is the least expression by which

$$(x^2 - x)(x^2 - 1)(x^2 + 7x + 6)$$

must be multiplied to make it a perfect square ?

48 TEST PAPERS IN ELEMENTARY ALGEBRA

5. Solve the equations :

$$(1) \frac{1-x}{2} = \frac{3+x}{3} - \frac{9+x}{4}.$$

$$(2) 2(x-1) - 3(y-3) = 12; \quad 5(x-1) + 3(y-3) = 9.$$

6. A man's average is x runs for his first p innings and $x+y$ runs for his total number $p+q$ innings. What was his average for his last q innings?

7. A number consists of four digits, the first and last digit are the same, and the digit in the ten place is three times the digit in the hundred place. Prove that the number must be divisible by 13.

8. The price of a wine being raised 10 shillings a dozen, 4 dozen now cost the same as 5 dozen cost before. Find the original price of the wine per dozen.

47.

1. If $2x - 3y = 1$ and $3x + 5y = 2$, find the value of $\frac{x}{y}$.

2. (1) Divide $x^4 + 4x + 3$ by $x^2 - 2x + 3$.

(2) If $y = \frac{1+x}{1-x}$ and if $x^3 = x^2 - 1$, prove that

$$y^3 + y^2 + 7y = 1.$$

3. In x years' time, A will be p times as old as B will be; B is now y years old. What is A 's present age? Give a full explanation.

4. Find the factors of

$$(1) x^5 + x^4 + x^3 + x^2.$$

$$(2) (3a-b)^2 - (2a+c)^2.$$

$$(3) a^2 - b^2 - c^2 + 2bc.$$

$$(4) 8x^4 - 18x^2 + 9.$$

$$(5) x^2 - (2a-3b)x - 6ab.$$

5. How many minutes is it before 12 o'clock, if 20 minutes ago it was three times as many minutes past ten?

6. Simplify (1) $\left(x + \frac{10}{x} - 7\right) \div \left(x - \frac{6}{x} + 1\right)$.

(2) $(2x)(-3xy)^2(-xyz^2)^3\left(\frac{1}{2}xy\right)^4$.

(3) $\left[\frac{1}{ab+b^2} + \frac{1}{ab-a^2}\right] \div \left[\frac{1}{a^2-b^2} - \frac{1}{2ab}\right]$.

7. Find two numbers such that one-tenth of the larger exceeds one-ninth of the smaller by 4, and one-eighth of the larger exceeds one-seventh of the smaller also by 4.

8. A man owns two houses, of which the rents differ by 15£ a year. In five years he receives 10£ more from the tenant of the more highly rented house than he receives in six years from the other tenant. What are the rents of the two houses?

48.

1. If $x = a + b$ and $y = a - b$, prove that $ax - by = bx + ay$ and that $3b(x^2 + y^2) - (x^3 - y^3) = 4b^3$.

2. Find the value of a if $x^2 + 2x + 1$ is a factor of

$$x^4 - 6x^2 - 8x + a;$$

and prove that in this case $x - 3$ is also a factor.

3. Find a number such that its fifth, seventh and ninth parts exceed its third part by 38.

4. Solve the equations:

(1) $\frac{x+1}{2} + \frac{x+2}{3} = 14 + \frac{5-x}{4}$.

(2) $(x-3)^2 = 16$.

(3) $2x + 3y - z = 1 = 4x - 2y + 3z = 6x + 7y - 5z + 8$.

50 TEST PAPERS IN ELEMENTARY ALGEBRA

5. The populations of two countries are a and b ; and these are at the rate of x persons and y persons to the square mile. Find the average population per square mile in the two countries taken together.

6. Simplify (1) $\frac{x^2 - 5x + 6}{x^2 - x - 6} \times \frac{3x^2 + 7x + 2}{3x^2 - 5x - 2}$.

(2) $\left[1 + \frac{1}{a} + \frac{1}{a(a+1)}\right] \div \left[1 + a + \frac{1}{a+1}\right]$.

7. Prove that $\frac{1}{x-a} + \frac{1}{x-b}$ has the same value when $x = a+b$ as when $x = \frac{2ab}{a+b}$.

8. A man wished to pay some people a shilling each, but found that he had sixpence too little; so he paid them ninepence each and then had ninepence left. How much money had he?

49.

1. Find in degrees the angle between the hands of a clock at x minutes past y o'clock. At what times are the hands of a clock at right angles between four and five o'clock?

2. If $x^2 = 3x + 5$, prove that $x^4 = 57x + 70$.

3. If the answer to the simultaneous equations

$$ax + by + cz = 4, \quad 2ax - by + 3cz = 13, \quad ax - 3by + 4cz = 14$$

is $x = 3$, $y = -1$, $z = 2$, find the values of a , b , c .

4. (1) Find the L.C.M. of

$$3x^2 + 6x + 3; \quad 3x^2 + 3x; \quad x^3 - x; \quad (x+1)(x+2)^2.$$

(2) Find the factors of

$$(1) x^4 - 5x^2 + 4.$$

$$(2) a(b^2 + c^2) - bc(1 + a^2).$$

TEST PAPERS IN ELEMENTARY ALGEBRA 51

5. B and C have between them 9£ more than A . If C gives half of what he has to B , B will have as much as A ; and if B gives half of what he has to C , C will have 1£ more than A . How much has each?

6. Solve the equations:

$$(1) (x+1)(x+2)(x+3) = (x+4)(x+5)(x-3).$$

$$(2) \cdot 2x - \cdot 3y = \cdot 6x + \cdot 5y = 7.$$

7. If you add 12 to a number and divide by 5, the result is the same as subtracting 2 and dividing by 3. Prove that the same result would be obtained by adding 33 and dividing by 8.

8. For what range of values of x is the expression

$$(x-1)(x-3)(x-5)$$

positive? For what range of values of x is the expression $x^2 - 4x + 3$ negative?

50.

1. (1) If $a = \frac{1}{1-b}$, $b = \frac{1}{1-c}$, prove that $c = \frac{1}{1-a}$.

(2) Prove that

$$3(x+1)^2(x+2) - 2(x+1)(x+2)^2 = (x^2-1)(x+2).$$

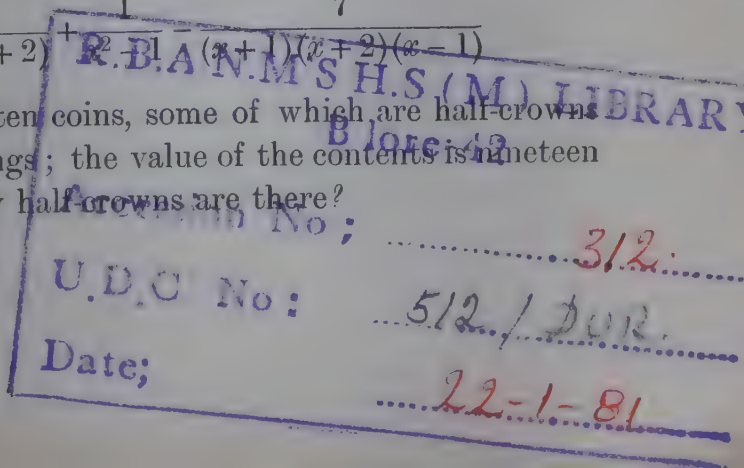
2. If a, b, c, d, e are five consecutive integers, prove that $ad + be - ac = c^2$.

3. Simplify

$$(1) \left(\frac{a}{2a-b} - \frac{a}{2a+b} \right) \times \frac{4a^2 - b^2}{a^2b + ab^2} \div \frac{1}{a^2 - b^2}.$$

$$(2) \frac{2}{(x-1)(x+2)} + \frac{1}{x^2-1} + \frac{7}{(x+1)(x+2)(x-1)}$$

4. A bag contains ten coins, some of which are half-crowns and the rest are shillings; the value of the contents is nineteen shillings. How many half-crowns are there?



52 TEST PAPERS IN ELEMENTARY ALGEBRA

5. Find the factors of

$$(1) a(a+c) - b(b+c). \quad (2) x^4 + x^2 + 1.$$

$$(3) (ab+cd)^2 - (bd+ac)^2. \quad (4) x^2 - xy - 6y^2.$$

6. For what value of k does the sum of $2(3k-1)$ and $3(2k+1)$ amount to $4(2k+3)$?

7. A cyclist riding at 12 miles an hour passes a train going in the opposite direction in $2\frac{1}{2}$ seconds. If he had been going in the same direction as the train, it would have taken $4\frac{1}{2}$ seconds to pass him. Find the length of the train in yards.

8. Find a fraction which reduces to $\frac{1}{4}$ when 1 is added to the denominator, and reduces to $\frac{2}{7}$ when 2 is added to the numerator.

PAPERS 51-60 include

Quadratics by factors.

51.

1. (1) If $x^2 = x + 1$, prove that $x^5 = 5x + 3$.

(2) Divide $x^5 - 5x - 3$ by $x^2 - x - 1$.

2. Simplify

$$(1) \left(\frac{a}{b} + \frac{b}{a}\right)\left(\frac{c}{d} + \frac{d}{c}\right) - \left(\frac{a}{b} - \frac{b}{a}\right)\left(\frac{c}{d} - \frac{d}{c}\right).$$

$$(2) \left(x + \frac{12}{x} - 7\right)\left(x + \frac{5}{x} - 6\right) \div \left(x + \frac{3}{x} - 4\right).$$

$$(3) \frac{1}{(x-1)(x-4)} - \frac{6}{(x-1)(x+2)(x-4)}.$$

3. If the sum of the first $2x$ prime numbers is not a prime number, what is the least possible integral value of x ? [1 and 2 are regarded as prime numbers.]

4. Solve the equations :

$$(1) x^2 = 4(2x - 3). \quad (2) \frac{x}{y} = 3 + \frac{1}{2y}; \quad \frac{y}{x} = \frac{3}{2x} - 1.$$

5. Prove the following rule : Multiply the price in farthings per lb. by $2\frac{1}{3}$, and you obtain the price in £ per ton.

6. Find the factors of

$$\begin{array}{ll} (1) 2x^3 - 50x. & (2) x^3 - y^3 + xy^2 - x^2y. \\ (3) 4x^2 + 28xy + 49y^2. & (4) x^2 - y^2 - z^2 + 2yz. \end{array}$$

7. A , B , C together have 88£; A has twice as much as C . If A gives 10£ to B , then B will have as much as A and C together. How much has each ?

8. In an election the Liberal majority was 207 and the number of voters was 4055. If there were only two candidates, how many must change sides at the next election in order that a Conservative may be returned ?

52.

1. If x is a positive fraction less than 1, prove that

$$1 - \frac{1}{2}x - \frac{1}{8}x^2$$

is greater than the square root of $1 - x$. Verify this if $x = \frac{1}{100}$.

2. (1) If $x = \frac{2y - 1}{3y - 2}$ and $y = \frac{2z - 1}{3z - 2}$, express x in terms of z .

$$(2) \text{ Simplify } \left(\frac{a + 2b}{a - b} - \frac{a - 2b}{a + b} \right) \div \left(\frac{6ab}{a^2 - b^2} \right).$$

3. Find the factors of

$$\begin{array}{l} (1) 6ac - 9bc + 3bd + 4ae - 2ad - 6be. \\ (2) (1 + x)^2(1 + y^2) - (1 + y)^2(1 + x^2). \\ (3) (3x - 2y)^2 - (x + 3y)^2. \end{array}$$

54 TEST PAPERS IN ELEMENTARY ALGEBRA

4. C is the mid point of AB and D is a point on AC ; $AB = 2x$ inches and $DC = y$ inches. Find, in terms of x and y , the values of $AD^2 + DB^2$ and $AB^2 - (AD^2 + DC^2 + CB^2)$.

5. Solve the equations :

$$(1) \frac{x-1}{x+2} = \frac{x+3}{x-4}.$$

$$(2) x(x-1) - 4(3x-4) = (3+x)(2-x).$$

$$(3) 3x - 8 = x - 2y - 1 = 40.$$

6. A man buys 350 eggs, some at 10 for a shilling and some at 9 for a shilling; he sells them at 1s. 6d. a dozen and gains 7s. 1d. How many of each kind did he buy?

7. Mr. Gladstone was $x-3$ years old in the year x^2 A.D. When was he born?

8. At a village concert the front seats are one shilling and the back seats sixpence. There are 250 seats altogether, and if all of these are occupied the receipts are £8. 15s. How many front seats are there?

53.

1. Find the result when the product of $2x^2 - 19x + 35$ and $2x^2 - 13x + 15$ is divided by $4x^2 - 16x + 15$.

2. Prove that the pair of values which satisfy the equations $2x - y = 4$, $x + 2y = 9$ also satisfy the equation $8x + y = 30$.

3. Find the factors of

$$(1) 12x^2 + 4x - 5. \quad (2) 4x^2 - (x - 3y)^2.$$

$$(3) 4x^3 + 8x^2 - x - 2. \quad (4) 6x^2 - 24.$$

4. Two lbs. of tea and three lbs. of coffee cost 9 shillings; if the price of the tea rises 25% and that of the coffee rises 20%, the total cost becomes 11 shillings. Find the price per lb. of each.

5. Solve the equations :

$$(1) \frac{x-2}{x+1} + \frac{x+2}{x-2} = 2.$$

$$(2) x(x-2) = 3(x+2).$$

$$(3) x^2 - 2y = 3, \quad x + y = 6.$$

6. $ABCD$ is a straight line such that BC equals CD . Prove that $AD^2 = AC^2 + BC^2 + AC \cdot BD$.

7. If $2 = \frac{1}{a} = a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{d}$, prove that $5abcd = 1$.

8. The combined age of three men is eighty ; eleven years ago the youngest was half the age of the eldest, and fourteen years ago the youngest was half the age of the second. Find the present age of each.

54.

1. If any three consecutive whole numbers are multiplied together, and if the middle number is added, prove that the result is a perfect cube.

2. Simplify

$$(1) \frac{x^2 + y^2}{x^2 - xy} \times \left(1 + \frac{x^2 - y^2}{x^2 + y^2} \right).$$

$$(2) \frac{bc}{(b-a)(c-a)} + \frac{ca}{(c-b)(a-b)} + \frac{ab}{(a-c)(b-c)}.$$

3. (1) Divide $x^4 + 4x + 3$ by $x^2 - 2x + 3$.

(2) For what value of x has the expression $\frac{x^2 + 1}{x - 1}$ the same value as it has when x equals 2?

4. Find a fraction which reduces to $\frac{3}{8}$ when 6 is added to the denominator, and reduces to $\frac{3}{10}$ when 6 is subtracted from the numerator.

56 TEST PAPERS IN ELEMENTARY ALGEBRA

5. Find the factors of

(1) $6 - x - x^2$.

(2) $x^2y^2 - 7xyz - 18z^2$.

(3) $(x + 2y)^2 + 7(x + 2y) + 10$.

(4) $(x + y)^2 + (xy - 1)^2$.

6. Solve the equations :

(1) $\frac{2x-1}{3} + \frac{4x+1}{5} + \frac{12-3x}{4} = 0$.

(2) $\frac{x-3}{x+3} + 2 = \frac{2x-1}{x-1}$.

(3) $3x - y = 4y + 2z = z + x = 10$.

7. A , B , C are three villages on a certain road ; X motors from A at 20 miles an hour, Y bicycles from B at 12 miles an hour and Z walks from C at 4 miles an hour. They start at the same time along this road in the same direction. When X overtakes Y , Z is 2 miles in front ; but if Z had only walked 3 miles an hour, he would only have been $\frac{3}{4}$ mile in front. Find the distance from A to B and from B to C .

8. The contents of a bag containing shillings, florins and half-crowns are worth 1£. 10s. ; there are 14 coins in all. Prove that the number of half-crowns exceeds by 4 twice the number of shillings.

55.

1. (1) Simplify

$$(x - y)^2 + (x + 2y)(2x - y) - \{x(3x - y) - y(3x + y)\}.$$

(2) If $x + y = 3$, prove that $x^3 + y^3 + 9xy = 27$.

2. Find the factors of

(1) $14x^2 - 13xy - 12y^2$.

(2) $3x^3 - 2x^2 - 3x + 2$.

(3) $x^4 - 10x^2 + 9$.

(4) $(ac - bd)^2 + (ad + bc)^2$.

3. The products of three consecutive whole numbers taken

in pairs are added together; unity is added and the result is divided by 3. Prove that the quotient is a perfect square.

4. Simplify

$$(1) \frac{x^2 + 6x + 8}{x^2 - 3x - 10} \div \frac{x^2 + 4x}{x^2 - 4x - 5}.$$

$$(2) \frac{1}{x^2} \left\{ (x-1) \left[1 + 1 \div \left(x + \frac{1}{x} \right) \right] + 1 \right\}.$$

5. The work done by x men in $x+4$ days is four-thirds of the work done by $x-1$ men in $2x-3$ days. Find the value of x .

6. Solve the equations:

$$(1) x - 1 - \frac{1}{2}(x-2) + \frac{1}{3}(x-3) = 0.$$

$$(2) 3x + y + z = \frac{1}{10}(6x + 3y + 2z) = \frac{1}{100}(8x + 4y + 3z) = 1.$$

$$(3) \frac{2x-1}{x+1} = \frac{3x-10}{x+3} + \frac{7}{8}.$$

7. At a fair, in shooting at a mark, a man receives three-pence if he strikes it and pays one penny if he misses it; he has 40 shots and has to pay fourpence. How many times did he hit the mark?

8. A man bought a number of handkerchiefs for 3£; if he had got three more for the same money, they would have cost one shilling less apiece. How many did he buy?

56.

1. Divide $9(3x-1)(x+1) + 2(4x+3)(x-1) + 12$ by $5x+3$.

2. Simplify

$$(1) \frac{3a}{3a-5b} + \frac{5b}{5b-3a}.$$

$$(2) \left\{ \frac{x^2}{x+y} - \frac{y^2}{x-y} \right\} \div \left\{ \frac{x}{x^2+y^2} - \frac{y}{x^2-y^2} \right\}.$$

58 TEST PAPERS IN ELEMENTARY ALGEBRA

3. In a ten-mile race A starts at x miles an hour and B at $\frac{1}{2}x$ yards per second; after five minutes, B is leading by 30 yards. Find x .

4. Find the factors of

$$(1) 9x^2 - 30x + 25. \quad (2) 9x^2 - 30x + 24.$$

$$(3) x^2 + xz - y^2 - yz. \quad (4) 1 + x + x^3 + x^4.$$

5. Solve the equations :

$$(1) \frac{1}{5}(4x - 3) - \frac{1}{8}(5x - 3) = 1. \quad (2) (2x + 1)^2 = (x + 5)^2.$$

$$(3) xy - 2x^2 = 3, \quad 3xy + x^2 = 72.$$

6. A rides from X to Y in 3 hours; B goes 4 miles an hour faster and takes 48 minutes less to get from X to Y . Find the distance of Y from X .

7. Prove that the value of $(x + 5)^2 + (x + 7)^2 - 2(x + 6)^2$ does not depend on the value of x ; and find the value of x for which the expression $(x + 5)^2 + (x + 4)^2 - 2(x + 6)^2$ is equal to zero.

8. A dinner was ordered for a certain number of people, the cost being £4. 10s.; as three failed to come, each person had to pay 1s. 6d. more. How many were present?

57.

1. If $4x + 2y = 7x + 6y = 10$, find the value of $x^3 + y^3$.

2. Simplify

$$(1) \left(\frac{a}{b} + \frac{b}{a} - 2 \right) \div \left(1 - \frac{2ab}{a^2 + b^2} \right) - ab \left(\frac{1}{a} - \frac{1}{b} \right)^2.$$

$$(2) \frac{bc}{b-c} + \frac{ca}{c-a} + \frac{ab}{a-b} + \frac{b^2c^2 + c^2a^2 + a^2b^2}{(b-c)(c-a)(a-b)}.$$

3. Find the factors of

$$(1) 1 - x - 2x^2. \quad (2) a^4 - 9b^2.$$

$$(3) a^2y^3 + a - ay^3 - a^2. \quad (4) (x + 2)^2(x + 3) - (x + 3)^2(x - 1).$$

4. If x pounds y shillings is doubled, the result is y pounds x shillings, where x, y are whole numbers less than twenty. Find x and y .

5. Solve the equations :

$$(1) \frac{3x-1}{2y+1} = \frac{2x+2}{y+2} = 4.$$

$$(2) x + 2y + 3z = 10; \quad x + y + z = 6; \quad x - y = y - z.$$

$$(3) \frac{12}{x+5} = 3 - x.$$

6. A motorist goes from A to B and back again in 3 hours 20 minutes; he goes out at 16 miles an hour and back at 24 miles an hour. How far is it from A to B ?

7. Prove that the difference between the square of the sum and the sum of the squares of two numbers is equal to half the difference between the square of the sum and the square of the difference of the two numbers.

8. A man bought some cattle for £150; if he had bought five more for the same sum, each would have cost 1£ less. How many did he buy?

58.

1. Simplify

$$(ay - bx)^2 + (bz - cy)^2 + (cx - az)^2 + (ax + by + cz)^2 \\ - (a^2 + b^2 + c^2)(x^2 + y^2 + z^2).$$

2. If $2(x+1)$ is greater than $7-x$, and if $5(2-x)$ is greater than $2(x-5)$ and if x is a whole number, find its value.

3. Find the L.C.M. of $x^2 - 2x$, $x^2 - 2x + 1$, $x^3 - x$, $x^2 - 3x + 2$.

4. Eggs are bought at x pence a score, and are sold at a profit of y per cent. How many are sold for a shilling?

60 TEST PAPERS IN ELEMENTARY ALGEBRA

5. Simplify (1) $\frac{a + \frac{1}{a} - 2}{a - \frac{2}{a} - 1} \times \frac{a + \frac{1}{a} - 2\frac{1}{2}}{2a + \frac{1}{a} - 3}$.

(2) $\left(1 + \frac{y}{x-y}\right) \div \left(\frac{x^3 - xy^2}{x^3 - y^3} \times \frac{1}{(x-y)^2}\right)$.

6. A man pays income tax at ninepence in the pound on part of his income and at fifteenpence in the pound on the rest; his total income is £2000 and the total tax is 95£. On how much of his income does he pay at the lower rate?

7. Solve the equations:

(1) $\frac{1}{3}(2x - y + 4) = \frac{1}{2}(3x + y - 1) = \frac{1}{4}(x + y - 5)$.

(2) $x + \frac{1}{x} = 2.5$.

8. The forewheel of a carriage makes 40 more revolutions than the hindwheel in 100 yards; and one wheel is 2 feet more in circumference than the other. Find the number of revolutions each makes in a mile.

59.

1. Divide $(3x^2 - 2x + 4)^2 - (2x^2 + x - 1)^2$ by $x^2 - 3x + 5$.

2. Prove that $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$; and use this result to express 85×164 as the sum of two squares.

3. From a certain number 7 is taken and the result is divided by 6; to the same number 14 is added and the sum is divided by 9. If the two quotients are equal, find the number.

4. Simplify

(1) $\left\{ \frac{(a+b)^2}{a^2} - \frac{(a-b)^2}{b^2} \right\} \div \left(1 - \frac{2ab}{a^2 - b^2} \right)$.

$$(2) \frac{a^2 - (b-c)^2}{(a+c)^2 - b^2} \times \frac{b^2 - (c-a)^2}{(a+b)^2 - c^2} \times \frac{c^2 - (a-b)^2}{(b+c)^2 - a^2}.$$

5. Solve the equations :

$$(1) \frac{1}{5} \left(\frac{1}{x} - \frac{1}{3} \right) + \frac{1}{7} \left(\frac{1}{x} - \frac{1}{5} \right) + \frac{1}{3} \left(\frac{1}{x} - \frac{1}{7} \right) = 0.$$

$$(2) 2x + y = 5a + b, \quad 2y - x = 7b.$$

$$(3) \frac{1}{x-7} + \frac{1}{2x-14} = \frac{1}{x-8}.$$

6. A man buys some articles at 16 for a shilling, and sells them for 1s. 2d. a dozen and makes a profit of £2. 10s. Find the number of articles.

7. A ladder 26 feet long rests against a wall with its foot at a distance of 10 feet from the wall. How much must the end be pulled out so that the top moves down through the same distance?

8. *A* who travels 8 miles an hour leaves a town *P* at the same time as *B*, *C* leave *Q* in opposite directions; *C* walks 4 miles an hour. When *A* meets *B*, *C* is 10 miles away; and *A* overtakes *C* when *B* is entering *P*. How far is it from *P* to *Q*?

60.

1. Prove that the two expressions $\frac{20}{x(x+3)}$ and $3x-4$ are equal when $x=2$ and when $x=-2$. What are their values when $x = -\frac{5}{3}$?

2. If $x = \frac{ab-c^2}{a-b}$, find in terms of a, b, c the value of

$$\frac{(x+a)(x-b)}{(ax+c^2)(bx-c^2)}.$$

62 TEST PAPERS IN ELEMENTARY ALGEBRA

3. Find the factors of

$$(1) x^2 - 17x + 72.$$

$$(2) 1 - 2a + 2ab - b^2.$$

$$(3) (ac + bde)^2 - e(ad + bc)^2.$$

$$(4) x^4 + 4y^4.$$

4. In a hollow square there are x men in each front rank, and the square is y men deep. How many men compose it? State one way in which 60 men could be arranged in a hollow square.

5. Solve the equations :

$$(1) \frac{2x+1}{6} = \frac{x-1}{x+1} + \frac{x+1}{3}.$$

$$(2) x + \frac{4}{x} = \frac{16x}{4+x^2}.$$

$$(3) x+y=5, \quad 2y+z=10, \quad 2z+3x=14.$$

6. If 18 is added to a certain number of two digits, the result is the same as subtracting 18 from twice the number formed by reversing the digits. And if 63 is added to the number, the result is the same as subtracting 63 from four times the number formed by reversing the digits. Find the number.

7. $ABCDE$ is a straight line such that C is the mid point of AE and of BD . Prove that $AE^2 + 2BC^2 = AD^2 + 2AC^2 + AB^2$.

8. A 's income is three times B 's income; each of them saves 400£ a year, and A spends five times as much as B . How much does A spend?

PAPERS 61-70 include

Quadratics by completing the square; functional notation; remainder theorem.

61.

1. If the sides of a triangle are x , y , z inches, it can be proved that the square on the diameter of the circle inscribed in the triangle contains $\frac{(x+y-z)(y+z-x)(z+x-y)}{x+y+z}$ square

inches. Find the radius of the circle inscribed in the triangle whose sides are (1) 3, 4, 5 inches, and (2) 4, 5, 6 inches.

2. If $x + y = 1$, prove that $x^3 + y(3x + 1) = y^3 + 2xy(x + 2y) + x$.

3. If $f(x) \equiv x^2 - 3x + 1$, find the values of $f(1)$, $f(0)$, $f(-1)$, and find a value of x for which $f(x - 1)$ and $f(2 - x)$ are equal.

4. In x years' time A will be twice as old as B ; y years ago A was three times as old as B , and z years ago A was four times as old as B . Find a relation between x , y and z .

5. Find the factors of

(1) $x^2 - y^2 - x - y$.

(2) $x^3 + 2x^2y + 3xy^2 + 6y^3$.

(3) $x^3 + x - 2$.

(4) $x^4 + 64y^4$.

6. Solve the equations:

(1) $\frac{1}{x+1} + \frac{1}{x-1} = \frac{2}{x+3}$.

(2) $\frac{1}{x+1} + \frac{2}{x-1} = \frac{1}{x+3}$.

(3) $\cdot 05(x - \cdot 3) + \cdot 2(x - \cdot 1) = \cdot 23$.

7. Find a number such that twice the difference between its fifth and seventh parts is less by 3 than its eighth part.

8. The number of fives balls that can be bought for one pound is equal to the number of shillings which 180 balls cost. Find the cost of a dozen fives balls.

62.

1. Prove that the expression $x^2 + y^2 + z^2 - yz - zx - xy$ is unaltered in value if x, y, z are all increased by the same amount.

2. The expression $ax^2 + bx + c$ is equal to 9, 2, 6 when x equals $-1, 0, 2$. Find its value when $x = 1$.

3. If $f(x) \equiv \frac{ax+b}{cx-a}$ and $x = f(y)$, prove that $y = f(x)$.

4. Simplify

$$(1) \left(\frac{2ab}{a-b} - b \right) \left(1 - \frac{2ab}{a^2+b^2} \right) \left(\frac{a}{b} - \frac{2b}{a+b} \right) \left(\frac{a}{b} - \frac{2a-b}{a+2b} \right).$$

$$(2) \frac{x+y}{y} - \frac{x}{x+y} - \frac{x^3 - x^2y}{x^2y - y^3}.$$

5. If 10 is added to the sum of the fourth powers of three consecutive integers, prove that the result is divisible by 3 and that the quotient thus obtained is a perfect square.

6. Solve the equations :

$$(1) \frac{x-2}{x-6} + \frac{x-7}{x-5} = 2. \quad (2) 2x + \frac{1}{y} = 10, \quad 3x - \frac{2}{y} = 1.$$

$$(3) \frac{3}{x+5} - \frac{1}{2x+3} = \frac{1}{4}.$$

7. A motorist who travels 9 miles an hour faster than a cyclist takes $1\frac{1}{2}$ hours less time to go 42 miles. Find their speeds.

8. What integral value of x makes $3x-1$ greater than $x+5$ and $4x-7$ less than $x+7$?

63.

1. If $a - \frac{1}{a} = x$ and $b - \frac{1}{b} = y$, prove that

$$\left(\frac{1}{b} + \frac{1}{a}\right)\left(\frac{1}{b} - \frac{1}{a}\right)(a^2b^2 - 1) = x^2 - y^2.$$

2. Find the factors of

(1) $45x^2 + 21xy - 6y^2$. (2) $(2a + b)^2 - 2a - b - 2$.

(3) $x^3 - 3x^2 - x + 3$. (4) $2x^3 + x^2 - 7x - 6$.

3. If $f(x) \equiv 2x^2 - 1$, find the value of

$$f(x+1) - 2f(x) + f(x-1).$$

4. Find in pounds the total cost of buying x articles at y for a shilling and y articles at x pence a dozen.

5. Solve the equations :

(1) $(x^2 - 3x + 2)(6x^2 + x - 2) = 0$. (2) $x^2 - 6x - 3 = 0$.

(3) $\frac{x+16}{x+17} + \frac{x+17}{x+16} = \frac{13}{6}$.

6. The denominator of a fraction exceeds the numerator by 5, and, if 4 is subtracted from the numerator, the fraction reduces to $\frac{6}{7}$. Find the fraction.

7. A man bought a certain number of articles at a certain number of shillings each ; if he had bought four less and paid 4 shillings more for each, the cost would have been 16 shillings more. How much less would the cost have been if he had bought four more and paid 4 shillings less for each ?

8. A man can walk a certain distance by taking 120 steps of equal length. If he steps 3 inches longer for one half and 3 inches shorter for the other half, he would take one step more. Find the distance in yards.

64.

1. (1) Divide $\frac{3x^2 - 2x - 1}{2x^2 - 3x + 1}$ by $\frac{3x^2 + x}{4x^2 - 1}$.

(2) Prove that $\{(x+y)^2 + x^2 + y^2\}^2 = 2\{(x+y)^4 + x^4 + y^4\}$.

2. (1) Find the value of a if $x^2 - 2x + 1$ is a factor of $x^6 - 6x + a$.

(2) Find the factors of $x^3 - x^2 - 5x + 6$.

3. Simplify

(1) $\frac{ab}{a+b-2 \div \left(\frac{1}{a} + \frac{1}{b}\right)} - \frac{a(a+b)^2}{a^2+b^2}$.

(2) $(a+b+c)(bc+ca+ab) - (b+c)(c+a)(a+b)$.

4. Separate 100 into two parts such that if one is divided by 9 and the other by 11, the sum of the quotients is 10.

5. If $f(x) = x^2$, prove that

$$f(x+y) = f(x) + f(y) + \frac{1}{2}[f(xy+1) - f(xy-1)].$$

6. Solve the equations :

(1) $(3x-10)^2 - 9(3x-10) - 22 = 0$.

(2) $\frac{x-1}{x-2} + \frac{x+1}{x+2} = 3$. (3) $3x^2 - 8x - 9 = 0$.

7. The sum of three numbers is 55, the least is one-quarter of the sum of the other two, and the greatest is 3 more than the sum of the other two. Find the numbers.

8. X, Y, Z are three places along a certain road, Z being 2 miles beyond Y ; A and B leave X at the same time; A walks to Y and back at 4 miles an hour; B walks to Z at $3\frac{1}{3}$ miles an hour and returns to X at 3 miles an hour; he arrives $2\frac{1}{3}$ hours after B . Find the distance from X to Y .

65.

1. Find the values of a, b if $x-1$ is a common factor of $2x^2 - (a-1)x - b$ and $3x^2 - (2a+1)x + 2b$.

2. Find the factors of

(1) $2x^3 - x - 1$. (2) $(1-x^2)(1-y^2) - 4xy$.

(3) $x^4 - 3x^2y^2 + y^4$.

3. Simplify

(1) $\frac{1}{x+y+\frac{1}{2}xy} + \frac{1}{x+y} \cdot \frac{1}{1+2\left(\frac{1}{x}+\frac{1}{y}\right)}$.

(2) $\left(\frac{x+1}{x+2} - \frac{x}{x+1}\right) \div \left(\frac{x-1}{x-2} - \frac{x}{x-1}\right) \times \frac{x^2-1}{x^2-4} - 1$.

4. In a certain railway carriage, if 4 people sit on each side they have 10 cms. more space than if 5 people sit on each side, Find the width of the whole seat.

5. If $f(x) = \frac{1}{1-x}$, prove that $f(f(x)) + f(1-x) = 1$.

6. Solve the equations :

(1) $\frac{1}{x-1} + \frac{1}{x-4} = \frac{2}{x-2}$.

(2) $4x - 6y - 3 = 7x + 2y - 4 = 3y - 2x + 24$.

(3) $\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3} = 0$.

7. A man walks from A to B at $3\frac{1}{2}$ miles an hour and rests half an hour at B . He then returns to A ; at first he runs at 7 miles an hour, but for the last half hour he walks at $3\frac{1}{2}$ miles an hour. His total time is $6\frac{3}{4}$ hours. How far is B from A ?

8. A takes 14 days longer than B to do a certain piece of work, which would take both of them together 24 days to perform. How many days would B require for it?

66.

1. If, when $x^2 + ax + b$ is divided by $x - 2$, the quotient is $x + 5$ and the remainder is 4, find the values of a , b .

2. If $a + b = 3$, $a - b = 5$, find the value of $\frac{a^2 + b^2}{a^3 - b^3}$.

3. Find the factors of

(1) $x^4 + x^3 + x + 1$. (2) $6x^2 - 4xy + 10yz - 15xz$.

(3) $(a + b + c)(bc + ca + ab) - abc$.

4. The perimeter of a rectangular field is a yards, and its length exceeds its breadth by b yards. Find the area of the field in terms of a , b .

5. If $f(x) \equiv x^2 - 2x - 5$, solve the equation $f(x) = f(x - 1)$, and find the value of $f(8) - 2f(4)$.

6. Find the speed of a train if an increase in speed of 3 miles per hour saves 10 minutes in 120 miles.

7. a , b and p , q are two pairs of consecutive whole numbers, prove that $bq - ap = b + q - 1$, if $a < b$ and $p < q$.

8. The perimeter of a rectangle is 32 inches; if a margin of two inches is cut off all the way round, the area would be reduced to one-fifth of its former amount. Find its area.

67.

1. If $2x + 3y = 4$ and $3x + 4y = 7$, prove that

$$x^3 + y^3 - 1 = 4(x^2 + y^2).$$

2. Simplify

(1) $\frac{3x - 2\frac{1}{2}}{3\frac{1}{3}} + \frac{2x - 1\frac{3}{4}}{1\frac{3}{7}}$.

(2) $\left(1 + \frac{2b}{a}\right) \div \left(1 - \frac{3b}{a}\right) \times \left(\frac{a}{b} - 3\right) \div \left[\{(a + b)^2 - b^2\} \left\{\frac{a}{b} + \frac{b}{a}\right\}\right]$.

3. (1) If $x - 3$ is a factor of $2x^2 + ax - 9$, what is the value of a and what is the other factor?

(2) If $f(x) = \frac{x+1}{2x-1}$, prove that

$$(1 - 4x^2)[f(x+1) - f(x)] = 3.$$

4. A man who earns $3x + 12$ pence an hour and works $x - 2$ hours a day receives 3 shillings more each day than a man who earns $2x + 4$ pence an hour and works $x + 1$ hours a day. Find the value of x .

5. Solve the equations :

(1) $2x = 1 \div \left[1 - \frac{1}{x+1} \right].$

(2) $\frac{5}{x} - \frac{3}{y} = \frac{7}{x} - \frac{6}{y} = 9.$

6. The ages of three people are 50, 28, 9; in how many years' time will the age of the oldest be equal to the sum of the ages of the other two?

7. If $a + b = 15$, $b + c = 16$, $c + d = 22$, find the value of $a + d$.

8. A goods train starts $\frac{3}{4}$ hour before an express, and completes a journey of 75 miles one hour after the arrival of the express, which travels 35 miles an hour faster than the goods train. How long did the goods train take over the journey?

68.

1. If $y = 2x + 1$, express $x^2 - 3xy - 2y^2$ in terms of x only; and prove that

$$\frac{2x}{y} = \frac{(y-1)(y-2)}{4x^2-1}.$$

2. Simplify (1) $\frac{5}{2x-10} - \frac{2}{3x-15} + \frac{7}{5x-25}.$

(2) $\left(x - 6 + \frac{12}{x+1} \right) \div \left(x - \frac{12}{x+1} \right).$

70 TEST PAPERS IN ELEMENTARY ALGEBRA

3. If $x+1$ and $x-2$ are factors of x^3+ax^2+bx-6 , find the values of a and b and the third factor.

Find the factors of

$$(1) 20 - pq - p^2q^2. \quad (2) (x - 2y - 3z)^2 - (3x + y)^2.$$

4. There are $3x+2y$ pints in one jar and $7x-y$ pints in another jar; $8x+19y$ pints are now added so as to make the amount in each jar the same. How much of this is put into the first jar?

5. Solve the equations:

$$(1) 1.2x - .7y = .1; \quad 1.23x + 1.1y = 9.19.$$

$$(2) x^3 + 6x^2 - 7x - 60 = 0. \quad (3) 2x^2 + 3x - 7 = 0.$$

6. A man trains from A to B at 18 miles an hour and walks back at 4 miles an hour; his journey occupies 4 hours 35 minutes. What is the distance from A to B ?

7. $ABCD$ are four points on a line; B is the mid point of AC . Prove that $DA^2 + DC^2 - 2DB^2 = \frac{1}{2}AC^2$.

8. If a cubical tank was made four feet deeper, one foot longer and two feet narrower, it would hold 64 cubic feet more. How much did it hold originally?

69.

1. If $y = x + \frac{1}{x}$ and $z = x - \frac{1}{x}$, prove that $y^2 - z^2 = 4$ and that $x^3 + \frac{1}{x^3} = y(z^2 + 1)$ and $x^3 - \frac{1}{x^3} = z(y^2 - 1)$.

2. (1) If $x-1$ is a common factor of x^2+ax+3 and $x^2+2ax+b$, find the values of a, b .

(2) If $f(x) = \frac{x^2+1}{x}$, prove that $f\left(\frac{x}{y}\right) = f\left(\frac{y}{x}\right)$.

3. Simplify

$$(1) \frac{2}{x^2 - 3x + 2} + \frac{6}{(x-1)(x-2)(x-4)}.$$

$$(2) \left(\frac{a+b}{a-b} - \frac{a-b}{a+b} \right) \div \left(\frac{2a+b}{a-2b} - \frac{2a-b}{a+2b} \right) - \frac{a^2 - 8b^2}{5a^2 - 5b^2}.$$

4. In a race of x miles, A defeats B by y yards; if B runs z feet a second, find A 's pace in miles per hour.

5. Write down the squares of

$$(1) x + 3y.$$

$$(2) 2x + \frac{5}{x}.$$

$$(3) x - 2y + 3z.$$

6. Solve the equations:

$$(1) \frac{1}{x-1} = \frac{x-5}{x^2+2}.$$

$$(2) a+b=17, \quad b+c=19, \quad c+d=21, \quad d-a=5.$$

7. A river runs 3 miles an hour; a man rows 12 miles up stream and back again in $7\frac{1}{2}$ hours. How long did the journey up stream take?

8. Three-fifths of a certain sum of money is invested at $4\frac{1}{2}$ per cent., one-quarter of it is invested at 6 per cent. and the remainder at 4 per cent.; the total income is 72£. Find the sum of money.

70.

1. What are the coefficients of x^3 , x^4 , x^5 in the product

$$(1 - 2x - x^3 - 7x^4)(3 + 5x + 2x^2 - 6x^4)?$$

2. (1) Divide $12x^4 + 2x^3 - 11x^2 + 1$ by $3x^2 + 2x - 1$.

$$(2) \text{Simplify } \frac{1}{3x^2-3} + \frac{1}{x+1} - \frac{2}{3-3x}.$$

72 TEST PAPERS IN ELEMENTARY ALGEBRA

3. Find the factors of

(1) $x^2y^2 + z^4 - x^2z^2 - y^2z^2$. (2) $2x^3 + 7x^2 - 14x + 5$.

(3) $(2a + b)^2 - 2a - b - 2$. (4) $(x + 3)(x - 1) - (2x - 3)(x + 1)$.

4. A man rows ten miles down stream in an hour and a quarter, and rows home again in three hours and twenty minutes. Find the pace of the stream.

5. Solve the equations :

(1) $\frac{20}{x+1} + \frac{24}{x+2} = 8$. (2) $\frac{1+x^3}{(1+x)^3} + \frac{1-x^3}{(1-x)^2} = \frac{22}{3}$.

(3) $2x + 3y - z = 11$, $7x - 5y + 11z = 22$, $y - z = 1$.

6. A number of two digits is equal to nine times the sum of its digits ; if, however, it is divided by the difference of its digits, the quotient is 11 and the remainder 4. Find the number.

7. Four pounds of tea cost as much as five pounds of coffee ; if the price of tea rises 10 per cent. and of coffee 25 per cent., what number of lbs. of coffee costs as much as five pounds of tea ?

8. A man buys $2x$ eggs for a shilling and sells them at $5x$ pence a dozen ; he makes eightpence profit on four dozen eggs. Find the cost price of a dozen eggs.

PAPERS 71-80 include

Detached coefficients ; very simple literals.

71.

1. (1) If $2s = a + b + c$, find the value of

$$(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2.$$

(2) If $x = b - c$, $y = c - a$, $z = a - b$, find the value of

$$x^3 + y^3 + z^3 - 3xyz.$$

2. Divide, using detached coefficients,

$$x^6 + 3x^5 + 4x^4 + 13x^3 + 9x^2 - 8x - 6 \text{ by } x^3 + 3x^2 - 2.$$

3. (1) If $x = 3b - 2c$ and $y = 2b + 5c$ and $b + 7c = 10$, what value of b will make x twice as much as y ?

- (2) If $v = u + ft$, find t in terms of u, v, f .

4. In paying two guineas a man uses x half-crowns, $3x - 4$ florins and $2x + 5$ sixpennies. Find x .

5. If $f(x) \equiv x^2 - 3x$, find the value of

$$f(x+1) + f(x-1) - 2f(x),$$

and solve the equation $[f(x)]^2 = 4x$.

6. A sum of sixteen guineas is divided equally among a certain number of boys. If there had been eight more boys, each would have received ninepence less. Find the number of boys.

7. A train travelling x miles an hour passes a telegraph pole in p seconds and a man walking in the same direction as the train at y miles an hour in q seconds. Prove that

$$\frac{p}{q} + \frac{y}{x} = 1.$$

8. Find x if the expressions $\frac{x^2 - 5x + 6}{x^2 + x - 6}$ and $\frac{x + 7}{x^2 - 49}$ are equal, and verify by substitution.

72.

1. (1) If $\alpha^5 = 1$ and $x = \alpha - \alpha^4$ and $\alpha \neq 1$, prove that

$$x^4 + 5x^2 + 5 = 0.$$

- (2) Find the value of $\frac{1}{x-2a} + \frac{2}{x+b} + \frac{1}{b}$ when $\frac{1}{x} + \frac{1}{3b} = \frac{1}{3a}$;
the answer must not contain x .

74 TEST PAPERS IN ELEMENTARY ALGEBRA

2. Find the factors of

(1) $x^2 - 4xy + 4y^2 - 3x + 6y + 2$.

(2) $(x-1)(x+1)(x+4)^2 - (x+1)(x+4)(x-3)^2$.

(3) $1 + x + y + z + xy + yz + zx + xyz$.

3. Solve the equations :

(1) $\frac{3x+8}{13} = \frac{2x+9}{10} - \frac{x+8}{2}$.

(2) $(x-a)(a-b) = (x+a)(a+b)$.

(3) $(x^2 - 2x)^2 - 2x(x-2) = 3$.

4. Find the remainder when $x^n + a^n + b^n$ is divided by $(x-a)(x-b)$.

5. If x cows and y sheep cost as much as p cows or q sheep, prove that $\frac{x}{p} + \frac{y}{q} = 1$.

6. A steamer goes 6 miles down a stream which flows at a uniform rate in $22\frac{1}{2}$ minutes, and a rowing boat makes the same journey in $1\frac{1}{2}$ hours; on the return journey the boat takes 7 times as long as the steamer. Find the pace of the stream and the time the rowing boat takes to return.

7. If $x + \frac{1}{y} = y + \frac{1}{z} = 1$, prove that $z + \frac{1}{x} = 1$.

8. The denominator of a fraction exceeds its numerator by 7, and the difference of the fraction and its reciprocal is $3\frac{1}{30}$. Find the fraction.

73.

1. Divide $\frac{x + \frac{1}{x} - 2}{x + \frac{1}{x} + 2}$ by the square of $\frac{x - \frac{1}{x}}{x + \frac{1}{x}}$.

2. Find the factors of

$$(1) 6x^2 - 10xy - 3xz + 5yz. \quad (2) bcd(a^2 - d^2) - a(b^2c^2 - d^4).$$

$$(3) (x+1)^3 - 4x - 4.$$

3. Solve the equations :

$$(1) \frac{30}{x+1} + \frac{36}{x-1} + 6 = 0.$$

$$(2) 2x + y - 3z = x + 3y - 4z = \frac{x-5z}{2} + y = 5.$$

If $y = \frac{7x-3}{3-2x}$, find x in terms of y .

4. By selling an antique chair for six pounds, which cost x shillings, a man gained $3x - 10$ per cent. Find the value of x .

5. Divide $x^5 - 3x^4 + x^3 - 4x^2 - 8x - 2$ by $x^3 + 2x + 2$, using detached coefficients.

Use detached coefficients to find the first four terms in the expansion of $(1+x+x^2+x^3)^3$.

6. A man bought a number of eggs at 15 a shilling ; six of them were afterwards broken and the rest were sold at tuppence a dozen. He lost threepence ; how many did he buy ?

7. A certain expression is exactly divisible by $x - 3$ and the quotient is $2x^2 + 7x - 4$. Prove that the expression is also divisible by $x + 4$, and find the quotient.

8. A man has to reach a certain place by a certain time. If he walks 4 miles an hour he will arrive 10 minutes late ; if he walks $3\frac{1}{2}$ miles an hour he will be 20 minutes late. How fast must he walk to arrive at the right time ?

74.

1. Prove that

$(ax + by + cz)^2 + bc(y - z)^2 + ca(z - x)^2 + ab(x - y)^2$
is equal to $(a + b + c)(ax^2 + by^2 + cz^2)$.

2. Use detached coefficients to expand
- $(1 - 2x - 3x^2)^2(1 + x)$
- , and divide the result by
- $1 + 2x + x^2$
- .

3. Find the factors of

$$(1) 2x^3 - x^2 - 13x - 6; \quad (2) 9x^4 + 31x^2y^2 + 12y^4;$$

and prove that $[a^2 - (b - c)^2][b^2 - (c - a)^2][c^2 - (a - b)^2]$ is a perfect square.

4. If
- x
- miles an hour is the same pace as 110 yards in
- x
- seconds, find the value of
- x
- .

5. If
- $V = \frac{1}{3}\pi r^2 h$
- , find (1)
- h
- in terms of
- V
- ,
- π
- ,
- r
- , and (2)
- r
- in terms of
- V
- ,
- π
- ,
- h
- .

6. Solve the equations:

$$(1) \frac{x-1}{.5} + \frac{2x+1}{.75} = 18. \quad (2) \frac{5x+2y}{9} = x+y = 2x+3y+1.$$

$$(3) 3x^2 - 7x = 1.$$

7. A man travelled 82 miles in 5 hours; part of it he cycled at 12 miles an hour, and the rest he motored at 20 miles an hour. How far did he bicycle?

8. A goods train leaves London at 1 p.m.; an express starts on the same route at 2.30 p.m. and passes the goods train 50 miles from London and is 5 miles ahead of it ten minutes later. Find their rates of travelling.

75.

1. If $2x + 3y = 1$, express $2x^2 + xy - 3y^2 + 3x + 4y - 1$ in terms of x .

2. If $f(x) = \frac{x-1}{x+1}$, find the value of $f[f(x)]$, and prove that $f(x) \cdot f\{f[f(x)]\} = -1$.

3. If a, b, c, d are four consecutive positive integers, of which a is the least, prove that $a + b + c + d = cd - ab$.

4. (1) Find the value of a if $x + 1$ is a factor of $ax^3 - (a-1)x^2 - (2a+1)x - a + 1$;
and in this case find the remaining factors.

(2) Prove that a is a factor of $(a+b+c)^4 - (b+c)^4 - (c+a)^4 - (a+b)^4 + a^4 + b^4 + c^4$;
hence put this expression into factors.

5. A man buys some pheasants at 5s. 6d. a brace; he sells three-fifths of them at 7s. a brace and the rest at 6s. a brace and clears £2. 15s. How many did he buy?

6. (1) If $I = \frac{P \times R \times T}{100}$, find R in terms of I, P, T .

(2) If $v^2 = u^2 + 2fs$, find u in terms of v, f, s , and find f in terms of v, u, s .

(3) Solve the equation:

$$\frac{3x-2}{2x-3} - \frac{2x-3}{3x-2} = 3\frac{3}{4}.$$

7. Find the value of x for which the expression

$$(x-1)(x+2) + 3x(x+3)$$

has the same value as it has when x equals 2.

78 TEST PAPERS IN ELEMENTARY ALGEBRA

8. An express train, which travels half as fast again as an ordinary train, performs a journey of 84 miles in 40 minutes less time. Find the speed of the express.

76.

1. (1) Divide $\frac{a+b}{a-b} + \frac{a-b}{a+b}$ by $\frac{a}{a+b} + \frac{b}{a-b}$.

(2) Find the coefficients of x^2 and x^4 in the expression $(3 - 2x - x^3 + 3x^4)(1 + 2x^2 + 3x^3 - x^4)$.

2. Find the factors of

(1) $6x^2 - 5xy - 6y^2$.

(2) $(a^2 - c^2)(a^2 - ac + c^2) + (a - c)(a^3c^3 + 1)$.

(3) $x^4 - 9x^2 + 12x - 4$.

3. (1) If $s = ut + \frac{1}{2}ft^2$, find f in terms of s , t , u .

(2) If $x = \frac{a+y}{b-c} = \frac{a-y}{b+c}$, find x in terms of a , b .

4. A man has enough money to buy p sheep or q cows. If he bought an equal number of sheep and cows, how many of each could he buy?

5. Solve the equations:

(1) $(x+1)(x+2) + (x+3)(x+4) = 2(x+1)(x+3)$.

(2) $8(x^2 - 1) = 2(x+1)^2 - 5x - 8$.

(3) $x(1 - 2y) = 27$, $x(7 + y) = 9$.

6. At an election, each of the electors voted for two out of the three candidates A , B , C . A received 23 votes, B 18 votes, C 19 votes. How many voted for both B and C ?

7. If a is so small a fraction that a^3 can be neglected, prove that $1 + \frac{1}{2}a$ is a root of the equation $x^3 + x^2 - x(1 + 2a) = 1$.

8. A man bought x articles for y shillings each. If he had bought 4 less and paid 4 shillings more for each, the cost would have been 16 shillings more. If he had bought 2 more and paid one shilling less for each, the cost would have been 2 shillings more. Find the values of x and y .

77.

1. Prove that $(x+y)(y-z) + (y+z)(z-x) + (z+x)(x-y)$
 $= \frac{1}{2}\{(x-y)^2 + (y-z)^2 + (z-x)^2\}.$

2. Find the values of a for which $x=1$ is a solution of the equation $(x+a)(2x+3a) - (x-a)(5x+3a) = 7$, and find the other values of x in this case.

3. Simplify (1) $\frac{\frac{x}{y} + \frac{y}{x} + 2}{\frac{x}{y} + \frac{y}{x} + 1} \times \frac{\frac{x^2}{y} - \frac{y^2}{x}}{\frac{x}{y} + 1} \div \left(1 - \frac{y^2}{x^2}\right).$

(2) $\frac{2x-7}{1 \cdot 2} + \frac{1-3x}{4 \cdot 2} - \frac{2x}{2 \cdot \frac{6}{11}}.$

4. A has $x\text{£}$ and B has $y\text{£}$. How must $z\text{£}$ be divided between A and B so that A may have three times as much as B in the end?

5. Solve the equations :

(1) $\frac{x+16}{x+17} + \frac{x+17}{x+16} = \frac{13}{6}.$

(2) $\frac{x-a}{x+b} + \frac{x-b}{x+a} = 2.$

6. A man pays income tax at one shilling in the pound on unearned income and ninepence in the pound on earned income. His earned income exceeds his unearned income by 10£, and the total tax he pays is 31£. Find his total income.

80 TEST PAPERS IN ELEMENTARY ALGEBRA

7. AD is a straight line bisected at C and produced to B so that $DB = \frac{1}{2}CD$; a point P is taken on AB such that $3AP^2 + PB^2 = AB^2 - AC^2$. Find the value of $\frac{AP}{AB}$.

8. From a number of three digits is subtracted the sum of the digits, and the result is divided by 9. If the quotient is 61 and if the original number was divisible by 17, find the number.

78.

1. If $\frac{a+x}{b+x} = \frac{b^3}{a^3}$ and $\frac{a+y}{b+y} = \frac{a^4}{b^4}$, prove that $xy = ab$.

2. Simplify

$$(1) \frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2}.$$

$$(2) \frac{2x^2 + 3xy - 2y^2}{x+y} \times \frac{x+3y}{2x^2 + 7xy - 4y^2} \div \frac{x^2 - 2xy - 8y^2}{x^2 - 2xy - 3y^2}.$$

3. If a, b, c are three consecutive integers, find the value of $a^2 + c^2 - 2b^2$.

4. Solve the equations:

$$(1) a(x-c) - c(x-a) = b(a-c). \quad (2) x^4 + 12 = 7x^2.$$

$$(3) \frac{6}{x} + 5y = 24; \quad \frac{9}{x} + 4y = 22.$$

5. Find by detached coefficients the expansion of

$$(1 - 2x + x^2 + x^3)^3$$

as far as x^2 .

6. A dealer buys a number of cameras at £4. 10s. each; he sells three-quarters of them at 6£. 10s. each, three of them at 6£ each and the remainder at 5£ each. His profit is 42£. Find the number of cameras.

7. If $\frac{(x-1)(x-2)}{2}$ and $\frac{(x+1)(x+3)}{5}$ represent consecutive integers, what integers are they?

8. A man travelled 65 miles in $3\frac{1}{2}$ hours; part of it he bicycled at $x-6$ miles an hour and the rest he motored at $x+2$ miles an hour. If he motored for $x+10$ minutes more than he bicycled, find the value of x .

79.

1. If $f(x) \equiv 2x^2 - 3x - 1$ and if $f(x) = f(x+1)$, find the numerical value of $f(x+2) - f(x-1)$.

2. What value of x makes the function $\frac{(x-1)(x-4)}{x-3}$ equal to (1) 0, (2) 2, (3) $-1\frac{1}{3}$?

3. Find a quadratic function of x such that, when it is divided by $x-1$, $x+2$, $x-2$, the remainders are -3 , -9 , $+3$ respectively.

4. A man buys x pounds of sugar for y pence; part of it cost p pence per lb. and the rest q pence per lb. How much did he buy of the first kind?

5. What must be added to the following in order to make them perfect squares: (1) $x^2 + 10x$; (2) $x^2 - 9x$; (3) $4x^2 + x$; (4) $25x^2 + 3x$?

Solve the equation $x^2 - 9x = 7$, giving each root correct to two places of decimals.

6. A man gave away 5£ more than one-quarter of the money he had and then had left 11£ less than four-fifths of what he had. How much did he give away?

7. The result of adding 2 to a certain number is twice as much as the result of subtracting 2 from its square. Find the number.

82 TEST PAPERS IN ELEMENTARY ALGEBRA

8. If the speed of a train is increased by 4 miles an hour, a saving of 15 minutes is made in a journey of 240 miles. How much more time would be required, if the speed was decreased by 10 miles an hour?

80.

1. If $2x + 3y = 5$, express $2x^2 + 3y^2$ in terms of x only; hence solve the equations $2x + 3y = 5$, $2x^2 + 3y^2 = 35$.

2. Simplify (1) $\left(x - \frac{6}{x+1}\right)\left(x + \frac{2}{x+3}\right) \div (xy + 2x - 2y - 4)$.

$$(2) \frac{3}{x+1} - \frac{3}{x+2} - \frac{1}{(x+1)^2} - \frac{2}{(x+2)^2}.$$

3. Prove that $x^2 + x + 1$ is a factor of $x^5 + x + 1$. Find the factors of (1) $(x+1)(x+2)^2 - (x+2)(x+3)^2$; (2) $9 - \frac{12}{x} + \frac{4}{x^2}$.

4. A man rides from A to B at x miles an hour and back again at y miles an hour; his total journey occupies z hours. How far is it from A to B , and how much time would he have saved if he had ridden the whole distance at $\frac{1}{2}(x+y)$ miles an hour?

5. Find the coefficients of x , x^3 , x^5 in the expression $(1 - 2x - 3x^2 - x^4 + 2x^5)(2 + x - 2x^3 + x^4 - x^5)$.

6. A takes 11£ more than B away with him for a holiday; at the end of the first week A has spent one-fifth of his money and B has spent one-eighth of his money. A has then got 7£ more than B . How much did A take away?

7. If the sum of two numbers is unity, prove that the sum of the first and the square of the second is equal to the sum of the second and the square of the first.

8. Find the whole number whose square added to its cube is nine times the next whole number.

PAPERS 81-90 include

*Simultaneous equations, one being quadratic;
undetermined coefficients.*

81.

1. If $x + y + z = 0$, prove that $2(x^4 + y^4 + z^4) = (x^2 + y^2 + z^2)^2$.
2. Prove that $x - 1$ and $x - 2$ are factors of

$$x^4 + x^3(a - 3) - x^2(3a - 5) + x(2a - 9) + 6$$
for all values of a . Find the value of a if $x + 1$ is also a factor, and in this case write down the remaining factor.
3. If $2x^2 - 3x - 5 \equiv a(x - 1)(x - 2) + b(x - 1) + c$ for all values of x , find the values of a , b , c .
4. Find the factors of
(1) $(a^2 + b^2)c + a(b^2 + c^2)$. (2) $(x + 1)^3 - (y - 1)^3$.
(3) $(x - 1)(x - 2)(x - 3) + 24$.
5. If x is the sum of a number and its reciprocal, and if y is the difference between the square of that number and the square of its reciprocal, prove that $(x^2 - 2)^2 - y^2 = 4$.
6. Solve the equations:
(1) $\frac{a}{bx} - \frac{b}{ax} = a - b$. (2) $x^2 - 3xy = 28$; $3x + 2y = 10$.
7. A , who walks x miles an hour, takes one hour longer to go z miles than B , who walks y miles an hour. Find z in terms of x and y .
8. In a motor car race between A and B , the former has half-an-hour's start; he covers the first half of the distance at 50 miles an hour, then stops for 15 minutes for repairs, and finishes at 40 miles an hour. B travels at 55 miles an hour

84 TEST PAPERS IN ELEMENTARY ALGEBRA

for the first half of his time and at 45 miles an hour for the remainder, making no stop. The result is a dead heat. Find the length of the course.

82.

1. (1) If $x = a + \frac{1}{b}$ and $y = b + \frac{1}{a}$, find in terms of a, b the value of $\left(\frac{1}{x} + \frac{1}{y}\right) \div \left(\frac{1}{a} + \frac{1}{b}\right)$.

(2) If $2s = a + b + c$, prove that

$$s^3 - (s - a)^3 - (s - b)^3 - (s - c)^3 = 3abc.$$

2. If $\lambda = (a - x)(b - c) = (b - x)(a - c)$, find λ in terms of a, b, c .

3. Using detached coefficients, divide $x^4 - x^3 - 5x^2 + 17x - 12$ by $x^2 + 2x - 3$, and multiply $x^3 - 7x + 3$, by $2x^3 - 3x^2 - 4$.

4. Simplify

(1) $\frac{2}{b(a-b)} + \frac{a-b}{b^2(a+b)} \left\{ 1 - \frac{a^2 - b^2}{a^2 - 2ab + b^2} \right\}.$

(2) $\left[\frac{1}{x-1} - \frac{1}{x+1} \right] \div \left[\frac{5}{x^2 - 2x + 1} - \frac{4}{x^2 + x - 2} \right].$

5. If $p(x^2 - 6xy^2) + qy^2(y - x) + y(x^2 + 4y^2)$ has $x^2 - 3xy + 2y^2$ as a factor, find the values of p, q .

6. A, B, C can walk x, y, z miles an hour respectively; one hour after A has started, B follows him, and one hour later C follows B . If B and C overtake A at the same place, prove that $\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$.

7. What integral value of x makes $3x - 1$ greater than $x + 5$ and also $4x - 7$ less than $x + 7$?

8. A path 10 feet wide runs round a rectangular lawn. If the area of the path is 3200 sq. feet, find the perimeter of the lawn.

83.

1. (1) Prove that $(a^2 - xb^2)(c^2 - xd^2) = (ac + xbd)^2 - x(ad + bc)^2$.

(2) If x, y, z are integers, each of which can be expressed as the difference of two squares, prove that xyz can be expressed as the difference of two squares.

2. Simplify (1) $\frac{x - 2y}{x^2 + 2xy - 3y^2} - \frac{x - 3y}{x^2 + xy - 2y^2}$.

$$(2) \frac{5}{x} - \frac{5}{x+1} - \frac{1}{x^2} - \frac{4}{(x+1)^2}.$$

3. Find the factors of

$$(1) (a+b)^2 - (c+d)^2 + (a+d)^2 - (b+c)^2.$$

$$(2) (2x+y)^2 - 2(2x+y)(x-3y) + (x-3y)^2.$$

$$(3) x^2 + \frac{10x}{yz} + \frac{21}{y^2z^2}.$$

4. If p, q, r are three consecutive primes to 3 [for example, 7, 8, 10; or 11, 13, 14, etc.], prove that $p(p-2q) - r(r-2q)$ has one of two constant values, and find them.

5. Solve the equations:

$$(1) \frac{x-7}{3} + \frac{2x-1}{5} = (x-9)^2 - (x-10)^2.$$

$$(2) 2x + y = 1, \quad x^2 - 2xy = 39.$$

$$(3) (x-a)(x-b) = (x-a-b)^2 + ab.$$

6. A man bought 40 apples of one kind and 60 of another for five shillings. If, with the same money, he had bought

86 TEST PAPERS IN ELEMENTARY ALGEBRA

all of one kind he would have got 30 more than if he had bought all of the other kind. What was the cost of each kind?

7. If $a(3x - 5) - 7$ is equal to $b(2x - 5) + 3$ for all values of x , find the values of a and b , and the value of the expressions.

8. A, B, C are three points on a line such that $AB = 5''$, $BC = 3''$; find the position of a point P on AC produced, such that

$$\frac{PA}{PB} + \frac{PC}{BC} = \frac{AC}{BC}.$$

84.

1. Find the coefficient of x^3 (1) in $(3x^2 - 7x + 1)(5x^2 + 2x - 8)$; and (2) in $(2x^3 - x^2 + 5x - 3)^2$.

2. Simplify (1) $\frac{7}{x^2 - x} + \frac{3}{x^2 - 2x} + \frac{5x + 1}{x^2 - 3x + 2}$.

$$(2) \left(\frac{a+b}{a-b} + \frac{a-b}{a+b} \right) \div \left(\frac{a}{b} + \frac{b}{a} \right).$$

3. Find the values of a for which $x = 2$ is a solution of the equation $(2x + a)(x + a - 1) - (x + a)(2x - a) + 5 = 0$.

4. A boy receives x marks for each sum he gets right and loses y marks for each he gets wrong; he does b sums and receives a marks. How many sums were right?

5. (1) If $ax(x - 1) + bx(x + 1) + c(x - 1)(x + 1)$ is equal to $x^2 + 6x - 3$ for all values of x , find the values of a, b, c .

(2) The value of $\frac{(x - y)^3 + (y - z)^3 + (z - x)^3}{(x - y)(y - z)(z - x)}$ does not depend on the values of x, y, z . What is the value of this fraction?

6. After a man has walked 3 miles more than one-quarter of his journey, he has still to go 8 miles more than one-fifth of his journey. Find the distance he has to go.

7. If $f(x) \equiv 2x + 3$, solve the equations

$$f(x+2) = f(2y-1), \quad f(x) \cdot f(y) = f(2y) + 2f(2x).$$

8. Two numbers differ by 8, and the sum of their squares exceeds their product by 169. Find the numbers.

85.

1. If the first x of the numbers 2, 5, 8, 11, 14, 17, ... add up to $ax^2 + bx$ for all values of x , where a and b are constants, find the values of a and b . Find also the sum of the first fifty numbers.

2. (1) By how much must $(2x^2 - xy - 3y^2)(2x^2 + 3xy + y^2)$ be multiplied in order that the result may be a perfect square?

(2) What must be added to $2x^4 - 5x^3 + 5x$ to make it divisible by $2x^2 + x - 1$? Find the factors of the expression obtained by adding this quantity.

3. Simplify (1) $\frac{5}{2-2x} - \frac{4}{3+3x} - \frac{3}{4x^2-4}$.

$$(2) \left(2n - \frac{16n-1}{64n-8}\right)^2 - \frac{1}{(64n-8)^2}.$$

4. $a+2b$ pounds of tea costing $2a-b$ pence per lb. are mixed with $2a-b$ pounds of tea costing $a+2b$ pence per lb. Find the cost of the mixture per lb.

5. (1) If $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, find u in terms of v, f .

(2) Solve the equations $x^2 + xy + 4y^2 = 2x - 2y = 4$.

88 TEST PAPERS IN ELEMENTARY ALGEBRA

6. A man has £2000 invested partly at 4 per cent. and partly at 6 per cent. ; his total income is 95£. How much is invested at 6 per cent. ?

7. $ABCD$ is a square sheet of paper ; E is the mid point of CD and $AB=8$ inches. The paper is folded so that the corner B falls on E . If the crease meets BC in K , find the length of BK .

8. Find two numbers such that the square of either exceeds the other by 21.

86.

1. Find an expression of the form $ax^2 + bx + c$ which takes the values 0, 0, 12 when x is 1, 2, 3 respectively.

2. Simplify

$$(1) \frac{1}{a^2 - b^2} + \frac{1}{(b - a)^2} - \frac{a}{(a + b)(a - b)^2}.$$

(2) $d(a - 2b) + c(b + e) - ae$ if a, b, c, d, e are consecutive integers, of which a is the greatest.

3. Find the factors of

$$(1) 27x^2 + 12xyz + y^2z^2. \quad (2) 16(2x - 1)^2 - 9(x + 3)^2.$$

$$(3) (ax + by)^2 + (bx - ay)^2.$$

4. If $f(x) = \frac{x}{1 - x}$, prove that

$$f[f(x)] = \frac{x}{1 - 2x} \text{ and } f\{f[f(x)]\} = \frac{x}{1 - 3x}.$$

5. Solve the equations :

$$(1) \frac{ax + \frac{1}{b^2}}{bx + \frac{1}{a^2}} = \frac{ax - 1}{bx - 1}. \quad (2) 7x - 2y = 5xy = 15x - 10y.$$

6. A train starts from X at 1 p.m. and is due at Y at 5 p.m. After travelling half the distance at the ordinary uniform rate it is delayed for half-an-hour by a breakdown on the line. The speed is then increased by six miles an hour, and it arrives at Y at 5.10 p.m. Find the distance from X to Y .

7. If $(3x-1)(2x+1)$ and $(2x-1)(3x+1)$ represent consecutive even whole numbers, what are their values?

8. Two parallel chords of a circle are of lengths x inches and y inches; the distance between them is z inches. Find the distance of each from the centre of the circle.

87.

1. Prove that $5(2x^2 + 2xy + y^2)(8x^2 + 2xy - 3y^2)$ is equal to $(3x+y)^4 - (x+2y)^4$.

2. (1) Find the L.C.M. of $x^2 + 7x + 12$, $x^3 + 3x^2$, $x^2 + 8x + 16$.

(2) Simplify $\frac{x(3+x)^2 - (1+3x)^2}{(1-x)^4}$.

3. If $2x^2 - 3x + 5$

$$= a(x-1)(x-2) + b(x-2)(x-3) + c(x-3)(x-1)$$

for all values of x , find the values of a , b , c . Check your answer by substituting for a , b , c .

4. A beats B by x yards in a mile race. If B takes y seconds longer to run z yards than A , how many minutes does B take to run a mile?

5. Solve the equations:

$$(1) (x+4)^3 = x^3 + 28.$$

$$(2) 3x + 4y = 18, \quad x^2 + 3xy + 2y^2 = 40.$$

6. If $x = 3 + \frac{7}{5+y}$, find y in terms of x , and prove that $(x-3)(y+7)$ equals $1+2x$.

90 TEST PAPERS IN ELEMENTARY ALGEBRA

7. A farmer has 70 more sheep than cows; after selling one-fifth of his cows and three-quarters of his sheep, he has ten more cows than sheep. How many sheep had he at first?

8. A cistern full of water weighs x cwt.; when two-thirds full it weighs y cwt. How much will it weigh when half full?

88.

1. Find the coefficients of x^2y^2 and xy^3 in
 $(3x^2 - 2xy - y^2)(5x^2 + xy + 2y^2).$

2. Simplify (1) $\frac{1}{x-1} + \frac{1}{x+1} + \frac{2x}{x^2+1}.$

(2) $\left(x + \frac{y-x}{1+xy}\right) \div \left(1 - \frac{x(y-x)}{1+xy}\right).$

3. If $x + y + z = 0$, express $2xy - 4yz$ in the form
 $ax^2 + by^2 + cz^2.$

4. A vessel which can hold $a + b + c$ pints contains a mixture of a pints of water and b pints of milk. It is filled up from another vessel which contains b pints of water and a pints of milk. How many pints of milk are there in the first vessel at the end?

5. If $f(x) = \frac{2x-3}{x+1}$, find the value of $f(x+1) - f(x-1)$, and prove that $f(x) + f\left(\frac{1}{x}\right) + 1 = 0.$

6. ABC is a triangle right angled at A ; AD is the perpendicular from A to BC and is of length 5 feet; BD and CD differ by 9 feet 11 inches. Find the lengths of AB , AC .

7. Express $x^2 + y^2 - 6x - 10y + 34$ as the sum of two squares. Hence find the values of x and y for which this expression is as small as possible.

8. A takes one hour longer to walk 16 miles than B takes to walk 15 miles; and A walks one mile in three minutes more than B does. How many miles an hour does B walk?

89.

1. Simplify (1) $\frac{1}{ac}\left(1 - \frac{a}{b}\right) + \frac{1}{ab}\left(1 - \frac{b}{c}\right) + \frac{1}{bc}\left(1 - \frac{c}{a}\right)$.

(2) $\frac{x+y}{y} - \frac{2x}{x+y} + \frac{x^3 - x^2y}{y^3 - x^2y}$.

2. If $p_1 = \frac{p_0}{1 + a_0 p_0}$, $p_2 = \frac{p_1}{1 + a_1 p_1}$, $p_3 = \frac{p_2}{1 + a_2 p_2}$,

prove that $p_3 = \frac{p_0}{1 + \lambda p_0}$, where $\lambda = a_0 + a_1 + a_2$.

3. Find the factors of

(1) $6x^3 + 13x^2 - 31x + 12$.

(2) $x^2 + 2 + \frac{1}{x^2}$.

(3) $x(y^2 + z^2) + yz(2x + y + z)$.

4. The line AD is divided at B, C so that $AB = \frac{1}{2}BC = \frac{1}{3}CD$; prove that $AC^2 = AB \cdot BD + BC^2$. If H is a point such that $BH = \frac{1}{4}BC$, find the value of $\frac{AH}{HD}$.

5. If $x^2 - 10x + 32$ is equal to $(x + a)^2 + b$ for all values of x , find the values of a, b ; hence find the least possible value of $x^2 - 10x + 32$, if x is real.

6. At an examination a candidate must obtain x per cent. of the total marks possible in order to pass. A obtains 28 per cent. and fails; B obtains 38 per cent. and passes. If A had obtained 160 more marks and B had got 240 marks fewer, each would have just passed. Find the value of x .

92 TEST PAPERS IN ELEMENTARY ALGEBRA

7. Solve the equations :

$$(1) (x-1)(x-4) + (x-5)(x-6) = 3(x+1) + 7.$$

$$(2) \frac{2}{x} + \frac{1}{y} = 7; \quad 3x - 4y = 1.$$

8. A sum of four guineas is divided equally among a certain number of persons; if there had been seven more, each share would have been one shilling less. How much more would each have received, if there had been seven fewer?

90.

1. If $x^4 - 6x^3 + 13x^2 - 12x + 4$ is equal to $(x^2 + ax + b)^2$ for all values of x , find the values of a and b .

2. (1) Multiply $1 - x + x^2 - x^3 + x^4 - x^5$ by $1 + x$.

(2) Multiply $x^2 - x^3 + x^4 - x^5 + \dots + x^{2n}$ by $1 + x$.

3. The H.C.F. of two expressions is $x^2(x-1)$ and their L.C.M. is $x^3(x-1)^2(x+2)(x-3)^2$; if one of them is $x^2(x-1)(x-3)^2$, find the other.

4. Solve the equations :

$$(1) \frac{3x+4}{5} - \frac{7x-3}{2} = \frac{x-16}{4}. \quad (2) \frac{x-a}{x-b} + \frac{x+a}{x+b} = 1.$$

$$(3) 7x - y = \frac{5}{x^2 + y^2} = 2.$$

5. A set of numbers is such that the n^{th} number is equal to $3n^2 - 5n$ for all values of n . Prove that the difference between the first and the seventh is equal to three times the difference between the third and the fifth number.

6. A man has a number of square blocks which he wishes to arrange in the form of a square; if he had 4 more or 21 fewer he could do it exactly; any other arrangement

would require more blocks or make use of fewer. How many blocks has he?

7. Prove that $x + y$ is a factor of $(x + y + z)(yz + zx + xy) - xyz$, and find the other factors.

8. A motor car takes $2\frac{3}{4}$ hours to go from A to B . It starts back again at the same pace, but after $1\frac{1}{4}$ hours it is delayed 15 minutes by an accident, and then finishes the journey at 12 miles an hour. The return journey occupies 4 hours. Where did the accident take place?

PAPERS 91-100 include

Square root; harder literals; H.C.F. by rule.

91.

1. If $2a = x + \frac{1}{x}$ and $2b = y + \frac{1}{y}$, find in terms of x and y the value of $ab \pm \sqrt{(1 - a^2)(1 - b^2)}$.

2. (1) Find a pair of values of x and y which satisfy the equation $(a + 3)x + (2 - a)y = 3a + 1$ for all values of a .

(2) If $a + b = 7$ and $ab = -2$, find the values of $a^2 + b^2$ and $\frac{a^2}{b} + \frac{b^2}{a}$.

3. Find the highest common factor of $4x^3 - 8x^2 - x + 2$ and $4x^3 - 12x^2 - 19x + 12$.

4. Find the values of b, c if $3x^2 + bx + c$ is the square root of $9x^4 - 24x^3 + 28x^2 - 16x + 4$.

5. If a, b, c are three consecutive integers, prove that $a^2 + b^2 + c^2 - (bc + ca + ab) = 3$.

94 TEST PAPERS IN ELEMENTARY ALGEBRA

6. Solve the equations :

$$(1) (x-a)(x-b) - (x-b)(x-c) - (x-c)(x-a) + x^2 = ab.$$

$$(2) x^2 - x\left(\frac{a}{b} + \frac{b}{a}\right) + 1 = 0. \quad (3) xy = 12z, \quad yz = 12x; \quad zx = 15y$$

7. The incomes of two men would become the same if one was increased and the other decreased by 4 per cent.; the sum of their incomes is 800£. Find the income of each.

8. A motorist who travels $12\frac{1}{2}$ miles an hour faster than a cyclist takes 50 minutes less over a journey of 15 miles. Find the speed of the cyclist.

92.

1. Find the values of a, b, c in order that

$$4x^6 - 24x^5 + ax^4 + bx^3 + cx^2 - 20x + 25$$

may be a perfect square.

2. If
$$\begin{aligned} x &= by + cz + du, & y &= ax + cz + du, \\ z &= ax + by + du, & u &= ax + by + cz, \end{aligned}$$

prove that
$$\frac{a}{1+a} + \frac{b}{1+b} + \frac{c}{1+c} + \frac{d}{1+d} = 1.$$

3. Factorise (1) $(x-y)^5 + (y-z)^5 + (z-x)^5$.

$$(2) (a+b+c)(ab+bc+ca) - abc.$$

$$(3) x^2 - 3xy + 2y^2 + y - x.$$

4. If $f(x) \equiv \frac{a^{2x} - 1}{a^{2x} + 1}$, prove that $f(2x) = \frac{2f(x)}{1 + [f(x)]^2}$.

5. (1) The sum of the digits of a number less than 100 is $3x + 5y$, the digit in the unit place is $4x + y$. What is the value of the number?

(2) x is a number of two digits. Prove that the sum of the digits of x and $8x$ is divisible by 9.

6. Solve the equations :

$$(1) 10xy = 5(2x + 5y) = 2(3x + 4y). \quad (2) x^2 - 2x = a^2 - 1.$$

7. A certain fraction reduces to p when one is added to the numerator and to q when one is added to the denominator and to r when one is added to both numerator and denominator.

Prove that $\frac{p}{r} = \frac{p+1}{q+1}$.

8. A luggage train leaves A at noon, and, travelling at a uniform rate, arrives at B at 5 p.m. A passenger train leaves B at 2 p.m., and, travelling uniformly, arrives at A at 4.30 p.m. The trains pass each other 40 miles from B . Find the distance from A to B , algebraically *and* graphically.

93.

1. If $x + y + z + w = 0$, prove that $(x + y)^2 + (x + z)^2 + (x + w)^2$ equals $x^2 + y^2 + z^2 + w^2$.

$$2. \text{ Simplify } (1) \frac{x^2 + 3xy + 2y^2}{x^2 + 2xy + y^2} \times \frac{x^2 + 5xy + 4y^2}{x^2 + 7xy + 12y^2}.$$

$$(2) \frac{5}{3x-6} - \frac{2}{7x-14} - \frac{4}{2-x}.$$

3. Prove that the difference of the cubes of two consecutive even numbers is greater by 2 than six times the square of the intervening odd number.

$$4. \text{ Find the square root of } 9x^4 - 42x^3 + 37x^2 + 28x + 4.$$

5. Solve the equations :

$$(1) x + y = 8; \quad \frac{x-y}{2} + \frac{2x-y}{3} + \frac{4}{3} = 0.$$

$$(2) ax - 2ay + bx = 2b^2; \quad ax - b(x - 2y) = 2ab.$$

6. A man bicycles five-sixths of the distance from X to Y at 15 miles an hour and walks the rest at 3 miles an hour ; he

96 TEST PAPERS IN ELEMENTARY ALGEBRA

took the same time as another man who travelled at a uniform rate. Find the pace of the latter.

7. The first n odd numbers add up to $an^2 + bn$, where a, b are the same for all values of n . Find the values of a and b .

8. In a 300 yards race A can beat B by x yards and can beat C by $\frac{1}{2}(10x+1)$ yards; B can beat C by $4x+1$ yards. Find the value of x .

94.

1. a, b are two consecutive integers, neither of which is divisible by 3. Prove that $a^2 + b^2 - 5$ is divisible by 36.

2. (1) If $xyz = 1$, prove that

$$\frac{1}{z(1+x+xy)} + \frac{1}{y(1+z+xz)} + \frac{1}{x(1+y+yz)} = 1.$$

(2) If $\frac{2}{x} + \frac{3}{y} = 4$ and $x + y = z$, find z in terms of x only.

3. Find the H.C.F. of

$$6x^4 - x^3 - 8x^2 - 4x - 3 \quad \text{and} \quad 2x^4 + 5x^3 + 4x^2 - 14x - 15.$$

4. The line AB is divided at C, D , such that $\frac{AC}{CB} = x$ and $\frac{AD}{DB} = y$. Find the value of $\frac{CD}{AB}$.

5. Solve the equations:

$$(1) \quad x - 5 = \frac{1}{7}(y - 2); \quad 4(y - 1) = \frac{1}{3}(x + 7).$$

$$(2) \quad (x + a)(x - a^3) + 4a^4 = 2a(x + a^3).$$

6. A man travelled seven-eighths of the distance from P to Q at 14 miles an hour and the rest at 3 miles an hour. He took 9 minutes more than a man who travelled the whole way at 10 miles an hour. How far is it from P to Q ?

7. Find the value of a if $x^2 - (a+2)x + 3a + 6$ is a perfect square.

8. At a party the ratio of ladies to gentlemen present was 7 to 5. But after 9 more ladies and 9 more gentlemen arrived, the ratio became 4 to 3. Find the total number present at the end.

95.

1. If a, b are consecutive even numbers, prove that $a^2 + b^2$ equals $2(ab + 2)$.

2. Simplify

$$(1) \frac{1}{x-1} - \frac{1}{2x+2} - \frac{x+3}{2x^2+2}$$

$$(2) \left\{ \frac{x^3 + x^2y + xy^2 + y^3}{x-y} - \frac{x^3 - x^2y + xy^2 - y^3}{x+y} \right\} \\ \left\{ \frac{x^2}{x^2y^2 + y^4} - \frac{y^2}{x^4 + x^2y^2} \right\}.$$

3. If $x + y + z = 0$, express $3xy + 4yz$ in the form $ax^2 + by^2 + cz^2$.

4. Solve the equations :

$$(1) \frac{x-a+b-c}{x+a-b-c} = \frac{b-c}{2a-b-c}.$$

$$(2) 1 + 2x = x^2 + y^2 = 2 + 3y.$$

5. A, B, C have 46£, 50£, 72£ respectively ; A gives B a certain sum ; B then gives C one-third of what he has. If C has now four times as much as A , how much did A give to B ?

6. The first five of a set of numbers are 1, 9, 23, 43, 69, and the n th number is of the form $an^2 + bn + c$. Find the numerical values of a, b, c .

7. If $2x+4$ and $3x-9$ are the squares of consecutive integers, find the value of x .

98 TEST PAPERS IN ELEMENTARY ALGEBRA

8. A, B, C are three stations on a line running due north ; X is 3 miles east of A and Y is 5 miles east of C ; the distance from A to C is 16 miles and the sum of the distances of X and Y from B is 18 miles. Find the distance from A to B .

96.

1. By what must the expression

$$(2x^3 - 3x^2 - 8x - 3)(6x^3 + x^2 + x + 1)$$

be multiplied in order to make it a perfect square ?

2. If $xy = a - 1$ and $z = y + 1$, prove that $(x - 1)(z - 1) = a - z$.

3. Find the factors of

$$(1) 4x^2 + 4xy - 3y^2 + 4x + 10y - 3.$$

$$(2) (x - 1)^2 + 5 + \frac{6}{(x - 1)^2} \quad (3) 3x^2 - 12y^4.$$

4. Find an integral value of x for which $4x^2 - 31x + 57$ is negative.

5. Find the values of a, b if $4x^4 - 4x^3 - 11x^2 + ax + b$ is a perfect square.

6. Find two numbers such that half the square of the first is three times the second, and one-third of the square of the second is sixteen times the first.

7. If $f(x)$ is a rational integral algebraic function of x having $x - a$ and $x - b$ as factors, prove that, if $f(x)$ is divided by $(x - a)(x - b)(x - c)$, the remainder is

$$f(c) \cdot \frac{(x - a)(x - b)}{(c - a)(c - b)}.$$

What is the remainder if it is first divided by $x - a$, the quotient divided by $x - b$ and the new quotient by $x - c$?

8. A cistern which holds 1800 gallons is filled in three-quarters of an hour by three pipes, one of which conveys

TEST PAPERS IN ELEMENTARY ALGEBRA 99

8 gallons less and the other 6 gallons more than the third each minute. How long would it take for the largest pipe to fill the cistern by itself?

97.

1. (1) Simplify $\frac{1}{x-1} - \frac{x-2}{x^2+2}$.

(2) Prove that $x^2(x^2 - 3y^2)^2 + y^2(y^2 - 3x^2)^2$ is a perfect cube.

2. If $x^2 - 9x + 6 = 0$, find the numerical value of $x^2 + \frac{36}{x^2}$.

3. Solve the equations :

$$y = \frac{x+2a-6}{ax-3x+a-2}, \quad x = \frac{y+2a-6}{ay-3y+a-2},$$

and discuss the case when $a = 1$.

4. Find the H.C.F. of

$$x^4 - 7x^3 + 6x^2 + 5x - 5 \quad \text{and} \quad 2x^5 - 5x^4 + 11x^3 - 8x^2 - 7x + 7.$$

5. If $x = -1$ and $x = 2$ are two roots of the equation

$$ax^3 - 3x^2 + bx + 2 = 0,$$

find the third root.

6. Find a number which exceeds three times its reciprocal by two.

7. C is a point on the line AB , such that $AB \cdot CB = AC^2$; prove that $AC \cdot CB = AC^2 - CB^2$.

8. If successive powers of $n^2 + 1$ are divided by $n^2 - n + 1$, prove that only six distinct remainders are obtained.

98.

1. If

$$12x^2 + 2xy - 2y^2 - 11x + 7y - 5 \equiv (4x + 2y + p)(3x - y + q),$$

find the values of p, q .

100 TEST PAPERS IN ELEMENTARY ALGEBRA

2. (1) Divide $x^4 - (a+5)x^3 + (5a+b+4)x^2 - (4a+5b)x + 4b$ by $x^2 - 5x + 4$.

(2) Find the square root of $x^6 - 4x^4 - 6x^3 + 4x^2 + 12x + 9$.

3. Find a symmetric function of x and y of the second degree which equals 11 when $x=1, y=2$ and equals 8 when $x=-1, y=1$.

4. Solve the equations :

$$(1) \frac{2x+7}{x+3} + \frac{3x-2}{3x+4} = \frac{12x+17}{4x+7}.$$

$$(2) x^3 - 27x + 54 = 0.$$

$$(3) (a+b)x + (a-b)y = (a+b)y - (a-b)x = a(a^2 + b^2).$$

5. A cyclist leaves A at 15 miles an hour at the same time as a motorist leaves B at 21 miles an hour. They pass each other $7\frac{1}{2}$ miles from a village which is half-way between A and B . How far is it from A to B ?

6. Express the number 837 in the form $a + 8b + 64c + 512d$, where a, b, c, d are whole numbers, each of which is less than 8.

7. If $(x-\lambda)(a+b) = (x+\lambda)(a-b) = a^2 - b^2$, express λ and x in terms of a, b .

8. A ladder, of length $a^2 + 1$ feet, rests against a wall with its foot at a distance of $2a$ feet from the wall. What distance must the foot of the ladder be pulled out, so that the top may move down through an equal distance?

99.

1. Find the L.C.M. of $x^2 - 1, x^2 + 2x - 3, x^3 - 7x^2 + 6x$.

2. Solve the equations :

$$(1) \frac{x+3}{x} + 4x = 14.$$

$$(2) \frac{ax-b}{a} + \frac{bx-a}{b} = \frac{a}{b} - \frac{b}{a}.$$

3. If $x^2 + ax - 5$ is the square root of $x^4 + 6x^3 - x^2 - 30x + 25$, find the value of a .

4. If $f(x) \equiv \frac{x}{1-x}$ and $\phi(x) \equiv \frac{x}{1+x}$, prove that

$$f[\phi(x)] = \phi[f(x)].$$

5. Simplify

$$(1) \frac{y}{2(x-y)} - \frac{y}{2(x+y)} - \frac{y^2}{x^2} - \frac{y^4}{x^2(x^2-y^2)}.$$

$$(2) \left(1 + \frac{1}{x}\right)(x-1) - \left(1 + \frac{1}{y}\right)(y-1) \text{ if } x = \frac{a}{a-b}, y = \frac{b}{a-b}.$$

6. If a, b, c, d are four consecutive integers of which a is the greatest, prove that $ab - cd = a + b + c + d$.

7. The ratio of the numbers of the sides of two polygons is $2:3$, and the ratio of the sums of their angles is $3:5$. Find the number of sides of each.

8. (1) A room is x feet long, y feet wide, z feet high. Find the cost in shillings of papering the four walls if the paper is p inches wide and costs q pence a yard.

(2) What will be the cost if r per cent. of the paper bought is wasted?

100.

1. If $3x^2 - 14xy + 8y^2 + 2x + 12y - 8 \equiv a^2 - b^2$,
 and if $a = 2x - 3y - 1$,
 find b in terms of x and y .

2. Solve the equations:

$$(1) \frac{x+1}{x+2} + \frac{x+4}{x+5} = \frac{x+2}{x+3} + \frac{x+3}{x+4}.$$

$$(2) ax - (a-b)y = b^2, \quad (a+b)x - by = a^2 - b^2.$$

R.B.A.H. M'S HIGH SCHOOL,

Bangalore.

102 TEST PAPERS IN ELEMENTARY ALGEBRA

3. The side of a cube is increased by x per cent. By how much per cent. is its volume increased?

4. Find the factors of

$$(1) a^2b^3 - a^2 - b^3 + 1. \quad (2) x^4 - 41x^2 + 400.$$

$$(3) 1 + x - 2x^2 - x^3 + x^4.$$

5. Simplify $\left(x - \frac{y^2}{x}\right)\left(\frac{1}{x} + \frac{x}{y^2}\right) \div \left(\frac{x^4}{y^2} - y^2\right),$

and express $\frac{\frac{1}{x} - \frac{b}{a}}{\frac{1}{x} + \frac{b}{a}}$ in terms of a, b if $a(1 - x) = b(1 + x).$

6. A set of numbers is such that the sum of the first x numbers is $2x^3 + 3x^2$ for all values of x . Find the numerical values of the first, the second and the fifth numbers.

7. After a man has walked two miles more than one-quarter of his journey, he finds he still has to go one mile more than three-fifths of his journey. What is the total distance?

8. A train leaves A at noon and arrives at B at 1.30 p.m.; another train leaves B at 12.20 p.m. and arrives at A at 1.20 p.m.; each train travels at a uniform rate. At what time do they pass each other?

PAPERS 101-110 include

Simple indices; variation; easy ratio.

101.

1. If $f(n) = x^n + \frac{1}{x^n}$, prove that

$$\sqrt{f(2) + 2} = f(1), \text{ and } f(1) + f(3) = f(1) \cdot f(2).$$

2. Determine values of a, b which make
 $4x^4 - 12x^3 + ax^2 + bx + 16$
 a perfect square.

3. (1) If $\frac{a}{a+b-c} = \frac{2}{5}$ and $\frac{b}{a+b+c} = \frac{3}{4}$, find the ratio of
 a to b and of b to c .

(2) If $3x + 4y = 4$ and $73x - 87y = 12$, find the value
 of $\frac{x}{y}$.

4. Simplify $16^{\frac{1}{2}}$; $8^{\frac{2}{3}}$; 3^{-2} ; $12^{\frac{1}{3}} \times 75^{\frac{1}{3}}$; $\left(\frac{64}{125}\right)^{\frac{2}{3}} \times \left(\frac{4}{25}\right)^{-\frac{3}{2}}$.

5. A solid homogeneous sphere 4 feet in radius weighs 40 lbs.; find the weight of a similar sphere of radius 6 feet.

6. Solve the equations:

(1) $x^2 - 2y = y^2 - 2x = 3$.

(2) $x^4 + (x-4)^4 = 1312$.

(3) $\frac{a-x}{a-bc} = \frac{b-x}{b-ac}$.

7. Prove that 5^n can always be expressed in the form $p^2 + q^2$, where p, q, n are positive integers, neither being zero.

8. In a race between a cyclist and a motorist, the former has an hour's start and loses by 3 minutes. He rides for the first half of the *distance* at 20 miles an hour and for the remainder at 18 miles an hour. The motorist goes 35 miles an hour for the first half of his *time* and the remainder at 25 miles an hour. Find the length of the course.

102.

1. (1) Prove that the product of four consecutive integers, if increased by unity, becomes a perfect square.

(2) AB is a straight line x inches long, C is its middle point; P is a point dividing AB in the ratio 3 : 5. Find the length of CP .

104 TEST PAPERS IN ELEMENTARY ALGEBRA

2. (1) If $x^2 = 2x + 1$, prove that $x^5 = x^3 + 24x + 10$.

(2) Given that when the expression $ax^2 + bx + c$ is divided by $x - 1$, $x - 2$, $x - 3$, the remainders are 1, 2, 4. Find the values of a , b , c .

3. If $\frac{a}{b} = \frac{b}{c}$, prove that $\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} = \frac{a^3 + b^3 + c^3}{a^2b^2c^2}$.

4. Solve the equations:

$$(1) 10(x + y) = 7xy; 3y - x = \frac{1}{10}xy.$$

$$(2) \frac{x}{x-a} - \frac{b}{x+a} = \frac{x^2 + b^2}{x^2 - a^2}.$$

$$(3) \frac{x^2 - 5x + 3}{3x^2 + 4x - 7} = \frac{x^2 + 5x - 3}{3x^2 - 4x + 7}.$$

5. The distance a stone falls varies as the square of the time, if the resistance of the air is not taken into account. If it falls 80 feet in the third second, find how far it falls in 3 seconds.

6. Simplify (1) $32^{\frac{2}{3}} \times 125^{\frac{2}{3}} \times 81^{\frac{3}{4}} \times 100^{-\frac{1}{2}} \times 144^{-\frac{3}{2}}$.

$$(2) \left(\frac{9}{16}\right)^{-\frac{1}{2}} \times \left(3\frac{3}{8}\right)^{\frac{1}{3}}.$$

7. A number which consists of two digits is equal to four times the sum of its digits and twice the product of the digits. Find the number.

8. Between two towns the road is level for half the distance; a man can go on his tricycle 4, 8 and 12 miles an hour uphill, on the level and down hill respectively. It takes him 1 hour 55 minutes to go and 1 hour 35 minutes to return. How much is up hill on the return journey?

103.

1. Find the values of a , b , c in order that

$(x+a)(3x^3 + 4x^2 - 14x - 18) - (x^2 + bx + c)(3x^2 - 2x - 7)$
may be equal to 1 for all values of x .

2. The straight line AB is divided at P in the ratio $x:y$ and at Q in the ratio $2:3$. Find the ratio of PQ to AB .

3. Simplify (1) $2^x \cdot 3^y \cdot 6^{x-y}$. (2) $12^x \div 2^{2x}$.

If $4^x = 32^3$, find the value of x .

4. Solve the equations:

$$(1) \frac{2(x+p)}{x-q} + \frac{x+q}{x-p} = 3.$$

$$(2) (x+2y-1)(2x+y-1) = (x-2y+3)(3x+2y-5),$$

$$2x+y-1 = 3x+2y-5.$$

$$(3) 3x^2 - 5x - 1 = 0.$$

5. If 3 is one of the roots of the equation

$$ax^2 - (4a+b)x + 2a+b = 0,$$

find the other root.

6. The difference of two integers is 9 and the difference of their reciprocals is $\frac{1}{4}$; find them.

7. The volume of a sphere varies as the cube of its radius. If a metal sphere, 2 inches in radius, weighs 4 lbs., find the weight of a sphere of the same material, which is 3 inches in radius.

8. What function satisfies the following conditions:

$$f(x) \cdot f(y) = f(x+y) \quad \text{and} \quad f(1) = 2?$$

104.

1. If $y^2 + z^2 = ayz$, $z^2 + x^2 = bzx$, $x^2 + y^2 = cxy$, prove that
 $a^2 + b^2 + c^2 - abc = 4$.

2. Simplify (1) $16^{\frac{3}{4}}$. (2) $27^{-\frac{2}{3}}$. (3) $20^{\frac{1}{2}} \cdot 5^{\frac{3}{2}}$.

$$(4) [x(x+1)^{-1} + (x-1)^{-1}] \left(x + \frac{1}{x}\right)^{-1}.$$

3. Factorise $(x-y-z)^3 + y^3 + z^3 - x^3$. Prove that $a^2 - bc$ is a factor of $abc(a^3 + b^3 + c^3) - b^3c^3 - c^3a^3 - a^3b^3$, and hence factorise it.

106 TEST PAPERS IN ELEMENTARY ALGEBRA

4. If $3x - 2y + 4z = x + 2y - 3z = 0$, find the ratio of $x : y : z$.
Find also the value of $\frac{2y - z}{3x + z}$.

5. If $\frac{x^2 + y^2}{2xy} = \frac{x + y}{x - y}$, prove that $\left(\frac{x + y}{x - y}\right)^2 = \frac{x}{y}$.

6. The hypotenuse of a right-angled triangle is $x^2 + 1$ inches, and one of its sides is $2x$ inches. If the square on its perimeter is 24 times the area of the triangle, find the value of x .

7. A takes x minutes longer to walk y miles than B takes to walk z miles; and B walks one mile in p minutes less than A does. How many miles does A walk in an hour?

8. If the x th number of a set of numbers is $7x - 3$, by how much does the x th number exceed the $(x - 1)$ th number?

105.

1. If the expressions $x^2 + 5x + b$, $x^2 + x - 3b$ have a common factor, find the value of b .

2. If $x = 9$, $y = 16$, write down the values of $x^{\frac{1}{2}} + y^{\frac{1}{2}}$; $(x + y)^{\frac{1}{2}}$; $x^{-\frac{1}{2}} - y^{-\frac{1}{2}}$; xy^{-1} ; $x^{\frac{3}{2}} - y^{\frac{1}{2}}x$.

3. If $f(x) = \frac{a + bx}{b + ax}$, prove that $f[-f(x)] = -x$.

4. If the ratio of $x + 5y$ to $3x + y$ equals $4 : 3$, find the ratio of $2x + 3y$ to $x - 2y$.

5. The sum of any number of terms of the series of numbers 3, 9, 15, 21, 27, ... varies as the square of the number of terms taken. Find the sum of ten terms.

6. Two clocks P , Q are together at noon; when it is y o'clock according to P , the clock Q is a minutes behind.

How much must Q be put on so that they may be together when it is $(y+z)$ o'clock according to P ?

7. If $x^2yz=12$, $xy^2z=6$, $xyz^2=18$, find the value of $x+y+z$.

8. A cyclist starts at 2 p.m. to keep an appointment at 4.30 p.m. After $1\frac{1}{4}$ hours he is delayed 15 minutes by a puncture, and then finds he must increase his speed by 3 miles an hour in order to arrive in time. Where did the puncture take place?

106.

1. (1) Simplify $\frac{4x^3}{x^4-1} + \frac{1-\frac{1}{x^2}}{x+\frac{1}{x}} - \frac{1+\frac{1}{x^2}}{x-\frac{1}{x}}$.

(2) Prove that $(2x-3y)^2 + (x-2y)^2 - 2(2x-3y)(x-2y)$ is equal to $(3x+y)^2 + 4(x+y)^2 - 4(3x+y)(x+y)$.

2. Express $2x^2 - 5x - 3$ in the form

$$a(x-1)(x-2) + b(x-1) + c.$$

3. $ABCD$ are four points on a straight line, such that the ratios $\frac{AB}{BC}$ and $\frac{AD}{CD}$ are equal; prove that $\frac{2}{AC} = \frac{1}{AB} + \frac{1}{AD}$.

4. Simplify $(.01)^{\frac{1}{2}}$; $(.008)^{-\frac{1}{3}}$; $(2\frac{1}{4})^{-\frac{3}{2}}$; 10° ; $(\frac{9}{16})^{-2}$.

5. Solve the equations:

$$(1) \frac{x+2}{x-1} - 3 = \frac{2}{x} \quad (2) \left(\frac{x-c}{a-b}\right)^2 = \frac{cx}{ab}$$

6. Find a symmetric function of a , b , c of the second degree which equals 6 when $a=b=c=1$ and equals 7 when $a=1$, $b=0$, $c=-1$.

108 TEST PAPERS IN ELEMENTARY ALGEBRA

7. The ages of two people are in the ratio 8:5. In 12 years' time they will be in the ratio 11:8. After how many years will they be in the ratio 4:3?

8. The greatest height a stone attains when thrown vertically upwards varies as the square of its initial velocity. When projected with a velocity of 48 feet a second it ascends 36 feet. With what velocity must it be thrown to rise 25 feet?

107.

1. Prove that $(2x - y + 3z)^2 + (x + 3y - 2z)^2 - (3x + 2y + z)^2 + 2(2x - y + 3z)(x + 3y - 2z) = 0$.

2. If $(x+1)^2$ is greater than $5x-1$ and less than $7x-3$, find the integral value of x .

3. A, O, B, C, D are five points in order on a straight line, such that O is the mid point of AC . If the ratios $\frac{OB}{OC}$ and $\frac{OC}{OD}$ are equal, prove that the ratios $\frac{AB}{BC}$ and $\frac{AD}{CD}$ are equal.

4. Solve the equations:

$$(1) 9^x = 27. \quad (2) 4^x = \frac{1}{8}. \quad (3) 3 \cdot 27^x = 9.$$

5. If $8x^3 - 36x^2 + ax + b$ is a perfect cube, find the values of a and b .

6. A man travelled x miles in p hours; part of it he walked at y miles an hour; and the rest he drove at z miles an hour. How far did he walk?

7. A coach goes at the rate of 3 miles an hour uphill, 6 miles an hour on the level and 8 miles an hour downhill. On a certain journey the lengths of the uphill, level and downhill parts of the road are in the ratio 2:3:4, and the

time taken over the journey is 3 hours 20 minutes. Find the time spent on the uphill portion.

8. If the area of the surface of a sphere of radius 3 inches is x square inches, what is the area of the surface of a sphere of radius one foot?

108.

1. For what integral value of x is $167x$ greater than $21x^2 + 330$?

2. Find the square root of

$$x^6 + 4x^5 + 10x^4 + 10x^3 + 5x^2 - 6x + 1.$$

3. If $2x - 3y = 13$ and $3x + 4y = 11$, find the value of

$$6x^2 - xy - 12y^2 + 5x + y.$$

4. The sides of a triangle are in the ratio $x:y:z$ and the perimeter is p inches. Find the length of each side.

5. Solve the equations:

$$(1) \frac{x+b}{a+b} + \frac{a}{b} = \frac{x^2+ab}{bx}.$$

$$(2) x + \frac{1}{y} = y + \frac{1}{x}; \quad x + \frac{2}{y} = 3.$$

6. Simplify (1) $(27^{\frac{2}{3}} + 9^{\frac{3}{2}}) \times 81^{-\frac{3}{4}}.$

$$(2) \frac{4^x \times 27^{x-y}}{6^{2x}}. \quad (3) \frac{x-y}{x^{\frac{1}{2}} - y^{\frac{1}{2}}}.$$

7. A train leaves A at noon and arrives at B at p o'clock; another train leaves B at q o'clock and arrives at A at r o'clock. Each train travels at a uniform rate. At what time do they pass each other?

110 TEST PAPERS IN ELEMENTARY ALGEBRA

8. Two blocks of wood are the same shape: the area of the surface of one is four times that of the other. What is the ratio of their volumes?

109.

1. If a_1, a_2, a_3, a_4 are positive fractions less than 1, prove that $(1 - a_1)(1 - a_2)$ is greater than $1 - (a_1 + a_2)$ and that $(1 - a_1)(1 - a_2)(1 - a_3)(1 - a_4)$ is greater than

$$1 - (a_1 + a_2 + a_3 + a_4).$$

2. A function of x of the third degree is divisible by $2x^2 + 3x - 2$; it is also divisible by $2x^2 - 7x + 3$. When divided by $x + 1$, the remainder is 36. Find the function.

3. The area of a right-angled triangle is 120 square inches, and the altitude divides the hypotenuse in the ratio 25:144. Find the lengths of the three sides.

4. Simplify (1) $5 \cdot 4^{3n+1} - 20 \cdot 8^{2n}$.

$$(2) [1 - (1 - \{1 + x\}^{-1})^{-1}]^{-1}.$$

$$(3) (x^2y^{-1})^{-2} \times (x^{-4}y^3)^{-1}.$$

5. Solve the equations:

$$(1) x + y = 6, \quad x^2y^2 + 2xy - 35 = 0.$$

$$(2) (x + 1)^4 = (x - 1)^4.$$

6. A man bicycles a certain distance against the wind at p miles an hour, and then returns home again at q miles an hour. What is his average pace?

7. The radius of a sphere is increased by x per cent. Find the increase per cent. of the area of its surface.

8. A motor bus is timed to run from A to B in 45 minutes; on a particular journey it breaks down at a certain place, and, after a delay of 5 minutes, proceeds at one-quarter of its

former rate, and arrives 35 minutes late. How soon after the start did the breakdown take place?

110.

1. Prove that

$$(a+b+c)(ax^2+by^2+cz^2)-ab(x-y)^2-bc(y-z)^2-ca(z-x)^2$$

is a perfect square.

2. Find the value of a if one root of the equation $x^2-(a-1)x-10=0$ is -4 , and in that case find also the other root.

3. a, b, c are any three consecutive terms in the following set of numbers: $(2+1), (2^2+2), (2^3+3), (2^4+4), (2^5+5), \dots$. Prove that $3b=2a+c+1$, where $a < b < c$.

4. Solve the equations:

$$(1) \ x^2-y^2+x-y=26; \ x+y=12.$$

$$(2) \ 9^{x-y}=27; \ 16^{2x+y}=64.$$

5. Find the ratio of $a:b:c:d$ if

$$a+b+c+d=0, \ 2a+3b-2c+d=0, \ 2b-2c+d=0.$$

6. If $(x-1)(3x+1)$ and $2x(x+2)$ represent consecutive integers, what integers are they?

7. A, B can do a piece of work in x days; B and C can do it in y days; and A can do z times as much work as B in any given time. How long would A and C take to do it?

8. Apart from the cost of coal, the expenses of running a certain steamer are £25 a day; the consumption of coal per hour is proportional to the cube of the speed, and, when the steamer takes 10 days for a certain voyage, the cost of the coal is £400. Find the total cost if the same voyage lasts x days.

PAPERS 111-120 include

*Logarithms; simple gradients.***111.**

1. (1) If $a + \frac{1}{b} = c$, $b + \frac{1}{c} = d$, $c + \frac{1}{d} = a$, prove that $2cd = 1$.

(2) If $x^2 + 2y = 3$ and $y^2 + 2x = 5$, prove that

$$\sqrt[3]{x+3} = \frac{2}{x-1}.$$

2. What value of x makes the ratios $(6x+5):(5x+1)$ and $(12x+7):(10x+1)$ equal?

3. Prove that $2(b-c)^4 + 2(c-a)^4 + 2(a-b)^4$ is a perfect square.

4. Find by logarithms the values, correct to three significant figures, of (1) $\frac{186.4 \times 27.05}{1963}$. (2) $\frac{.0073 \times 1.08}{145}$.

5. The shape of a hill is obtained by drawing the graph of the function $\frac{1}{5}x(x+1)$, the unit being one inch for each axis. Find, by drawing and calculation, the slope of the hill at the place determined by $x=5$.

6. Simplify

$$(1) \frac{b+c}{(a-b)(a-c)} + \frac{c+a}{(b-a)(b-c)} + \frac{a+b}{(c-a)(c-b)}.$$

$$(2) \frac{x}{x^2-y^2} + \left(1 - \frac{y}{x} - \frac{x}{y}\right) \div \left(\frac{x^2}{y} + \frac{y^2}{x}\right).$$

7. The incomes of A and B are in the ratio of 5 to 4; their expenditures are in the ratio of 6 to 5, and their savings are in the ratio of 10 to 7. Find the ratio of A 's income to B 's expenditure.

8. The number of men required to form a hollow square is eight times the number required to fill the interior of it. Prove that the number of men in each face of the hollow square is three times the number of ranks.

112.

1. Simplify

$$(1) \left(\frac{x^3 + y^3}{x - y} - \frac{x^3 - y^3}{x + y} \right) \div \left(\frac{x}{y} + \frac{y}{x} \right).$$

$$(2) \frac{bc}{(b-a)(c-a)} + \frac{ca}{(c-b)(a-b)} + \frac{ab}{(a-c)(b-c)}.$$

2. If $f(x) \equiv 3x^2 - 5x + 4$,
find the value of $f(x-2) + f(x) - 2f(x+2)$,
and solve the equation $f(x) = f(x-1)$.

3. Solve the equations :

$$(1) \frac{x+10}{7} - \frac{2x+3}{5} + \frac{4x+1}{15} - \frac{x+3}{14} = 0.$$

$$(2) 2x^2 + y^2 = 3; \quad x + y = 2.$$

$$(3) x(1+y) = y(1+a) = a(1+x).$$

4. A man bicycles a certain distance against the wind at 8 miles an hour, and then returns home again at 15 miles an hour. What is his average pace?

5. (1) Find by logarithms, correct to three significant figures, the value of $\sqrt{.1784} \div [3.14 \times (.087)^3]$.

(2) Find, to three significant figures, the value of x if $10^x = 5$.

6. A motor starts from A for B , which is 45 miles away ; after $1\frac{1}{2}$ hours it breaks down and is delayed for 10 minutes and then proceeds at three-quarters of its former rate, and

114 TEST PAPERS IN ELEMENTARY ALGEBRA

consequently takes 25 minutes longer over the journey than it would otherwise have done. Find the time it actually takes.

7. Draw the graph of the function $\frac{1}{5}x(x-4)$, and read off its minimum value; calculate the value of the gradient (1) when $x=3$, (2) when $x=a$; hence obtain the value of a for which the gradient is zero.

8. If
$$\frac{\lambda}{a+x} + \frac{\mu}{b+x} + \frac{\nu}{c+x} + \frac{\lambda\mu\nu}{(a+x)(b+x)(c+x)} = 0$$
 for all values of x , find λ, μ, ν in terms of a, b, c .

113.

1. (1) If $u = a^2 - x^2 = b^2 - y^2 = (a+b)^2 - (x+y)^2$, find u in terms of a, b .

(2) If $x+y+z=2$, $x^2+y^2+z^2=3$, $x^3+y^3+z^3=4$, find the value of xyz .

2. Prove that $(x-1)^2$ is a factor of $mx^{m+n} - (m+n)x^m + n$.

3. If $\frac{a}{b} = \frac{c}{d}$, prove that $\left(\frac{1}{a} - \frac{1}{b}\right) - \left(\frac{1}{c} - \frac{1}{d}\right) = \frac{(a-b)(a-c)}{abc}$.

4. Find, correct to three significant figures, the values of

$$(1) \frac{.345 \times .068}{18.63 \times \frac{3}{4}} \quad (2) \sqrt{.8654} \times \sqrt[3]{.7512}.$$

5. A, B and C agree to share equally a hotel bill. If A pays x shillings and B y shillings and C z shillings, how much must A and B pay C to square accounts?

6. Given that the volume of a sphere varies as the cube of its radius, find the radius of sphere which is the same size as the three spheres, radii 5 inches, 7 inches, 8 inches, taken together. [Answer correct to $\frac{1}{10}$ inch.]

7. A man walks from A to B , which is 16 miles distant. Two hours after starting he increases his pace by 1 mile an

hour; he returns from B to A , walking $\frac{1}{2}$ mile an hour faster than the pace at which he arrived at B , but after two hours he reduces his speed by one mile an hour. If the return journey takes half-an-hour less than the out-journey, find the pace at which he left A .

8. Prove that the difference between the square of a number of two digits and the square of the number with the digits reversed is always divisible by 99.

114.

1. Factorise

$$(1) (a^2 - b^2)(a - b)^2 + b(2a + b)^3.$$

$$(2) (x + y + z)^3 - (y + z - x)^3 - (z + x - y)^3 - (x + y - z)^3,$$

and express $(2x + 3y)(x - 2y)$ as the difference of two squares.

2. Simplify

$$(1) \left(1 + \frac{y}{x}\right)^x \cdot \left(1 - \frac{x}{y}\right)^y \div \left\{\left(1 + \frac{x}{y}\right)^x \cdot \left(\frac{y}{x} - 1\right)^y\right\}.$$

$$(2) x^2 - px \text{ if } x = \frac{1}{2}\{p + \sqrt{p^2 + 4q}\}.$$

3. Find, correct to three significant figures, the values of

$$(1) (.0683)^{\frac{1}{3}}. \quad (2) 68.5 \times 3.72 - 10.4 \times \sqrt{1874}.$$

$$(3) \text{ A fourth proportional to } 71.4, 96.3, 124.$$

4. If $f(x) \equiv \frac{1}{2}(3^x + 3^{-x})$, express $f(2x)$ in terms of $f(x)$.

5. Find two consecutive integers the squares of which can be expressed by $(x - 2)(2x + 4)$ and $\frac{x}{2}(4x - 3)$.

6. A particle moves in such a way that it travels $3t + t^2$ feet in t seconds. How far does it travel (1) in the first 2 seconds; (2) in the first $2 + h$ seconds? What is its average

116 TEST PAPERS IN ELEMENTARY ALGEBRA

speed during the interval 2 seconds $\rightarrow 2 + h$ seconds? What is its speed after 2 seconds?

7. If the ratio of the volumes of two spheres is $a:b$, what is the ratio of the areas of their surfaces?

8. P, Q are two houses 5 miles apart; A leaves P at 12 o'clock and rides along the road which passes Q at 8 miles an hour; B leaves Q at 12.15 and walks along this road away from P at 4 miles an hour. When will A be 3 miles ahead of B ?

115.

1. If $x + y + z = xyz + 2$, prove that $(1 - xy)(1 - xz)$ is a perfect square.

2. Solve the equations:

$$(1) \frac{x-1}{x+1} + \frac{x-3}{x+3} = \frac{2(x-2)}{x+2}.$$

$$(2) \frac{3x+5y+6}{11} - \frac{x-2y}{18} = 7\frac{1}{9}, \quad \frac{7x-12y}{21} - \frac{x-y}{55} + \frac{1}{11} = 0.$$

$$(3) x = \frac{1}{y} = \frac{2}{x-1}.$$

3. If a, b, c are the lengths of the sides of a triangle, and if s is its semi-perimeter, the area is represented by

$$[s(s-a)(s-b)(s-c)]^{\frac{1}{2}}.$$

Find the area of the triangle, the sides of which are 3.75, 4.82, 5.07 inches, correct to three significant figures.

4. The ratio of two numbers is $2:3$; when a is added to each the ratio becomes $3:4$. What must now be added to each to make the ratio $4:5$?

5. Find a pair of values for x and y which satisfy the equation $(3 - 2a)x + (a + 4)y = 6 + 7a$, for all values of a .

TEST PAPERS IN ELEMENTARY ALGEBRA 117

6. (1) Express as powers of 8 the following: 2, 4, 16, $\frac{1}{2}$, $\cdot 125$, 32, $\sqrt{2}$.

(2) Find the least integral value of x for which 3^x is greater than a million.

7. What is the gradient of the function $3x^2 - 4x + 1$ when x equals 1? What is the least value of the function? Illustrate this by a rough figure.

8. ABC is a triangle right angled at A ; BC exceeds AB by 3 inches, and AB exceeds twice AC by 6 inches. Find the lengths of the three sides.

116.

1. For what range of values of x is the expression

$$(2x + 1)(x - 2)(x^2 - 15x + 50)$$

negative? Draw a *rough* figure to represent the graph of the function.

2. If $\frac{7a^2 - 4b^2}{a^2 - 2b^2} = \frac{4}{7}$, find the ratio of a to b .

3. (1) Solve, correct to two significant figures, $7^{x+1} = 4^{2x-3}$.

(2) Find, correct to three significant figures, a mean proportional to 83.64 and 37.51.

4. The formula for the sum of the first n numbers in the following series $1^3, 2^3, 3^3, 4^3, 5^3, \dots$ is $(an^2 + bn)^2$, where a, b are numbers which do not depend on the value of n . By taking special values for n , find the values of a, b . Use your result to find the value of $11^3 + 12^3 + 13^3 + \dots + 20^3$.

5. If x £ per ton is the same price as y lbs. for a shilling, find the numercial value of xy .

6. If $ax - by = 1$ and $ax^2 - by^2 = \frac{1}{a-b}$, prove that

$$ax^3 - by^3 = \frac{1}{(a-b)^2}.$$

7. The cost of issuing a print is partly fixed and partly varies as the number of copies. If the fixed cost is 15£, and each copy costs half-a-crown, what is the least number that must be sold at 12 shillings each to produce a profit?

8. If the price of oranges is lowered by a penny a dozen, two more would be obtained for a shilling. Find the price.

117.

1. Prove that the roots of the equation $2x^2 - 3x = 1$ exceed by 2 the roots of the equation $2x^2 + 5x + 1 = 0$.

2. For what other value of x has the expression $\frac{x^2 + a^2}{x - a}$ the same value as it has when x equals b ?

3. Find, correct to three significant figures, the value of

$$(1) \frac{3}{4} \times 3.142 \times (4.72)^3. \quad (2) \frac{5[1 - (\frac{4}{5})^{15}]}{1 - \frac{4}{5}}.$$

4. Express n in the form

$$\{a(n+1)^2 + b(n+1)\} - \{an^2 + bn\}.$$

5. If $f(x) \equiv ax^2 + bx + c$, where a, b, c are constants, prove that

$$a = \frac{1}{2}[f(x+1) + f(x-1) - 2f(x)]$$

and

$$b = \frac{1}{2}[f(x+1) - f(x) + f(-x) - f(-x-1)].$$

6. After a man has walked p miles more than the x th part of his journey, he finds he still has to go q miles more than the y th part of his journey. What is the total distance he has to go?

7. The sum of any number of terms of the set of numbers 2, 14, 38, 74, 122, 182, ... varies as the cube of the number of terms taken. Find the next number after 182, and find the sum of the first ten numbers.

8. A number of two digits is equal to nine times the sum of the digits. Prove that the number formed by reversing the digits is equal to twice the sum of the digits.

118.

1. Find a symmetric function of x, y of the third degree which equals 2 when $x=0, y=1$, and equals 24 when $x=1, y=2$.

2. Find the values of a, b if the equations $x^2 + 2bx + 4 = 5a$ and $4x^2 - 5ax - 4bx + 4b = 16a + 12$ have the same roots; and in this case find the roots.

3. If the ratios of $(x+y):(y+z):(z+x)$ equal 17:18:25, prove that the triangle whose sides are x, y, z is right angled.

4. (1) Find the least integral value of n for which $(.99)^n$ is less than .1.

(2) If $a=5, r=-.9, n=20$, find, correct to three significant figures, the value of $\frac{a}{1-r}(1-r^n)$.

5. Find the gradient of the function $x - \frac{x^3}{4}$ when x equals 1 and when $x=a$. Draw the graph of the function between $x=-3$ and $x=+3$.

6. A corrects proofs at the rate of a pages in p hours and B at the rate of b pages in q hours. If they both work together, how long will they take over a book of c pages?

120 TEST PAPERS IN ELEMENTARY ALGEBRA

7. Solve the equations :

$$(1) \frac{20}{x+10} - \frac{10}{x+20} = \frac{15}{x+5} - \frac{5}{x+15}.$$

$$(2) \frac{a}{x} - \frac{b}{a} = \frac{b}{x} - \frac{a}{b}.$$

8. If the sum of two numbers is three, prove that the sum of their cubes, when divided by nine, is equal to the excess of three over their product.

119.

1. If $x(1-x)$ equals $y(1-y)$, does it follow that x equals y , and if not, what is the other alternative? Give a numerical example.

2. Find the factors of

$$1-x+\frac{x(x-1)}{2}-\frac{x(x-1)(x-2)}{6}+\frac{x(x-1)(x-2)(x-3)}{24},$$

and prove that

$x-2$ is a factor of $x^3+(a-1)x^2-(a+5)x-2(a-3)$ for all values of a .

3. If the radius of a sphere is r inches, its volume is $\frac{4}{3}\pi r^3$ cubic inches. The outside diameter of a spherical shell is 8.32 inches and its volume is 63.2 cubic inches. Find its thickness, correct to one-tenth of an inch.

4. The ratio of the volumes of two cones is 4:5 and the ratio of the diameters of their bases is 4:15. Find the ratio of their heights.

5. (1) If $f(x) \equiv x+2^x$, prove that $f(x+1)-2f(x)=f(0)-x$.

(2) Divide 4^{2x} by 2^x .

6. (1) What is the equation whose roots are 1, -1, 0?

(2) If $4x^2+axy+25y^2$ is a perfect square, what is the value of a ? Is there more than one answer?

7. A and B are travelling in the same direction at 10 and 4 miles an hour respectively ; A reaches X one hour after B has passed Y , which is 17 miles away. How soon after leaving X will A overtake B ?

8. If a, b, c are consecutive odd integers, prove that 12 is a factor of $a^2 + b^2 + c^2 + 1$.

R. B. A. N. M.'S HIGH SCHOOL,

120.

Bangalore.

1. If $m = a^x$, $n = a^y$, $(m^y n^x)^z = a^2$, find a relation between x, y, z .

2. (1) Simplify $\left(\frac{x+y}{x-y} + \frac{3x^2-y^2}{x^2-y^2}\right) \div \left(\frac{3x-y}{x+2y} - \frac{3x(x+y)}{x^2-4y^2}\right)$.

(2) Prove that $x = 2 + \sqrt{3}$ is a root of the equation $x^4 + 15 = 56x$.

3. Solve the equations :

(1) $x + 2y + xy + 2 = 0$; $2x - xy = 9$. (2) $x^3 + 3 = 10x$.

4. The length in cms. of a seconds pendulum is given by the formula $\frac{gt^2}{\pi^2}$, where $g = 981$, $\pi = 3.142$, $t = 1$. Find the length.

5. Find the factors of

(1) $a(2bcy + ab^2) + y^2(c^2 - b^2)$. (2) $x^3 + x^2 - 2$.

(3) $4x^2 + 4xy - 3y^2 + 4x + 10y - 3$.

6. If a, b, c are consecutive integers, prove that the sum of their squares equals $3ac + 5$.

7. A bill for five guineas is paid in florins and half-crowns ; three more florins than half-crowns are used. Find the total number of coins.

8. Two blocks are the same shape and made of the same

122 TEST PAPERS IN ELEMENTARY ALGEBRA

materials ; the total areas of their surfaces are 18 square inches and 50 square inches. The weight of the smaller is 3 lbs. ; find the weight of the other.

PAPERS 121-130 include

Ratio and proportion ; indices and surds ; easy arithmetical progressions.

121.

1. If $a + b + c = 0$, prove that $a^2 - bc = b^2 - ca = c^2 - ab$. Prove also that $a^3 + b^3 + c^3 = 3abc$.

2. Solve the equations :

$$(1) (x+3)(x+4)(x+5) = 60.$$

$$(2) yz^2 = 75, \quad zx^2 = 20, \quad xy^2 = 18.$$

$$(3) \frac{(x+a)(x+b)}{x+a+b} = \frac{(x+c)(x+d)}{x+c+d}.$$

3. (1) If $\frac{a}{b} = \frac{c}{d}$, prove that $\frac{a^2}{a^2 - b^2} = \frac{a^2 - c^2}{(a^2 - b^2) - (c^2 - d^2)}$.

(2) If $\frac{x}{3} = -\frac{y}{2} = \frac{z}{7}$, find the value of $\frac{x+2y}{3y+z}$.

4. In a cross-country race a man averages 9 miles an hour for one-third of the distance ; he covers the next mile at 5 miles an hour, and finishes at 7 miles an hour ; his total time is 57 minutes. What is the length of the course ?

5. Subtract $\frac{x^n}{(x+y)^n}$ from $\frac{x^{n-1}}{(x+y)^{n-1}}$, and find the square root of $37 - 20\sqrt{3}$.

6. Two casks A and B contain mixtures of wine and water, A in the ratio of $7:2$ and B in the ratio of $4:1$. In what ratio must liquid be drawn from each cask to give a mixture in the ratio of $15:4$.

7. What is the first number in the n th bracket of the following series :

$$[1] + [2 + 3] + [4 + 5 + 6] + [7 + 8 + 9 + 10] + \dots$$

Find also the sum of the series inside this bracket.

8. If a number and its square root contain respectively p and q digits, prove that $2p - 4q + 1 = (-1)^p$.

122.

1. (1) Simplify $(1 - ax)(1 - bx)(1 - ax - bx)$ if $x = \frac{2}{a+b}$.

(2) If $s = a + b + c$, prove that $(as + bc)(bs + ca)(cs + ab)$ is a perfect square.

2. (1) If $\frac{a}{b} = \frac{c}{d}$, prove that $b(a + b - c - d) = (a + b)(b - d)$.

(2) If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$, prove that $b^2c + ad^2 = abd + c^3$.

3. Solve the equations :

(1) $xy = ax + by = bx + ay$.

(2) $\frac{x+y}{3} = \frac{y+z}{5} = \frac{z+u}{8} = \frac{u-x}{4}$; $3z + u - 2x - 4y = 8$.

4. Two trains starting from London and Edinburgh respectively, 400 miles apart, at the same time meet each other 250 miles from London, after travelling a number of hours equal to the excess of speed in miles per hour of one train over the other. Find the speed of each train.

5. Simplify

$$(1) \sqrt{2} \times \sqrt[3]{2} \times \sqrt[6]{2}. \quad (2) \sqrt{8} - 3\sqrt{2} + \sqrt{50} - \sqrt{16 + 16}.$$

$$(3) (2\sqrt{3} - 3\sqrt{2} - 5)(2\sqrt{3} + 3\sqrt{2} - 5).$$

6. The n th term in an arithmetical progression is $2n - 5$. Find the sum of the first 12 terms.

7. A man buys 7 oxen and 8 sheep for 94£; if the price of an ox falls 10 per cent. and that of a sheep $33\frac{1}{3}$ per cent., the value of his purchase falls 15£. Find the cost of 2 oxen and 3 sheep.

8. Prove that the difference between the square of the average of the n quantities $a_1 + x, a_2 + x, \dots, a_n + x$, and the average of their squares, is independent of x .

123.

1. If $a^2 - a\sqrt{3} + 1 = 0$, prove that $a^3 + \frac{1}{a^3} = 0$.

2. Simplify (1) $\left(1 + \frac{2a}{2x+a}\right)\left(2 - \frac{a}{x+a}\right)^2\left(x - a + \frac{2a}{\frac{2x}{a} + 3}\right)$.

$$(2) \frac{2\sqrt{2} - \sqrt{7}}{(2\sqrt{2} + \sqrt{7})^2} + \frac{2\sqrt{2} + \sqrt{7}}{(2\sqrt{2} - \sqrt{7})^2}.$$

3. If $\frac{a}{b} = \frac{b}{c}$, prove that $(a^2 + b^2)(b^2 + c^2) = b^2(a + c)^2$.

4. If $x + y + z = 5$, $x^2 + y^2 + z^2 = 11$, $x^3 + y^3 + z^3 = 18$, find the value of xyz .

5. The n th term of a series is $5n - 3$. Prove that the series is an arithmetical progression, and find the sum of n terms.

6. The ratio of A 's income to B 's expenditure is 10:11, and of B 's income to A 's expenditure is 15:7, and of A 's

savings to B 's savings is $3:4$. If B spends 200£ a year more than A , how much does A save?

7. If $n^2 \equiv [a(n+1)^3 + b(n+1)^2 + c(n+1)] - [an^3 + bn^2 + cn]$ for all values of n , find the numerical values of a, b, c . Hence find the value of $1^2 + 2^2 + 3^2 + \dots + 8^2 + 9^2$.

8. The straight line AB is bisected at C and produced to any point D . Prove that $AD \cdot BD = CD^2 - AC^2$.

124.

1. Prove that $2\{(b+c)(c-a) + (c+a)(a-b) + (a+b)(b-c)\}$ can be expressed as the sum of three squares.

2. Solve the equations:

$$(1) \quad x^2 + y^2 = 4y + 1 = 3 - 5x.$$

$$(2) \quad \frac{2(x+a)}{x-b} + \frac{x+b}{x-a} = 3.$$

3. If $\frac{a}{b} = \frac{b}{c}$, prove that $\frac{a^2 + ab + b^2}{b^2 + bc + c^2} = \frac{a}{c}$.

4. What number must be added to each of the numbers 13, 18, 41, 53 so as to make them in proportion?

5. In a certain series in A.P. the 8th term is twice the 4th term. Prove that the 9th term is three times the third term.

6. Simplify $\sqrt{x + \frac{1}{x} + 2} - \sqrt{x + \frac{1}{x} - 2}$.

7. The radius of the circle circumscribing a triangle whose sides are a, b, c is $\frac{1}{4}abc[s(s-a)(s-b)(s-c)]^{-\frac{1}{2}}$, where $s = \frac{1}{2}(a+b+c)$. Find, correct to $\frac{1}{10}$ inch, the radius of the circle circumscribing a triangle whose sides are 17.3, 21.5, 31.2 inches.

126 TEST PAPERS IN ELEMENTARY ALGEBRA

8. A man who is walking from A to B , a distance of $22\frac{1}{4}$ miles, meets at the end of 4 hours a man who has been walking for 3 hours from B to A . Were they each walking one-quarter of a mile per hour faster, they would meet a quarter of an hour sooner. At what rates are they walking?

125.

1. (1) Multiply $2^x - 3^y$ by $3^x + 2^y$.
(2) Solve $2^{x^2} = 16^{x-1}$.
2. Find a quadratic function of x which takes the values 1, 0, 2 when x equals 0, 1, 2 respectively.
3. Determine the value of c if zero is a root of the equation $\frac{x-3}{x-4} + \frac{x-4}{x-3} = \frac{2x-c}{x-5}$; and in this case find the other root.
4. If $\log 2 = .3010$ and $\log 3 = .4771$, show how to find, without using tables, the logarithms of (1) 50; (2) 18; (3) 1.5; (4) $\frac{4}{9}$; to three significant figures.
Evaluate to three significant figures $3.142 \times (.862)^2 \times 30.01$.
5. The sum of the first four terms of an arithmetical progression equals the sum of the first eight terms, and the sum of the first three terms is 27. Find the 20th term.
6. Three lbs. of tea and ten lbs. of sugar cost 9s. 4d.; if the price of tea rises 25 per cent. and that of sugar falls 20 per cent., the combined cost is 10s. 2d. Find the cost of each per lb.
7. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, prove that
$$(a^2 + b^2 + c^2)(x^2 + y^2 + z^2) = (ax + by + cz)^2.$$
8. A cricketer, on getting out in the first innings of a match, finds that he has by that innings increased his average

for the season by 2 ; on getting out in the second innings he finds he has still further increased his average by 2. In the whole match he made 100 runs ; how many did he make in the first innings ?

126.

1. Simplify

$$(1) \left\{ \left(\frac{x}{y} \right)^6 - \left(\frac{y}{x} \right)^6 \right\} \left\{ \frac{x}{y} - \frac{y}{x} \right\} \div \left\{ \left(\frac{x}{y} \right)^2 - \left(\frac{y}{x} \right)^2 \right\} \left\{ \left(\frac{x}{y} \right)^3 - \left(\frac{y}{x} \right)^3 \right\}.$$

$$(2) \frac{a^2 + b^2 + c^2 + 2ab + 2bc + 2ca}{a^2 - b^2 - c^2 - 2bc}.$$

2. Solve the equations

$$(1) (x+1)(x-1) + x(x-2) = (2x-1)(x-3).$$

$$(2) x^2 + y^2 = 34, \quad x - y = 8.$$

$$(3) \frac{3}{1 + \sqrt{x}} + \frac{3}{1 - \sqrt{x}} = 4.$$

3. If the n th term of a series of numbers is $3n + 5$, find the difference between the n th term and the $(n+1)$ th term.

Find also the sum of the first n terms.

$$4. \text{ Solve the equations } \frac{x}{10} = \frac{y}{14} = \frac{z}{35} = \frac{1}{10x + 14y + 35z}.$$

5. If $f(x) \equiv \frac{2}{a^x + a^{-x}}$, prove that

$$\frac{f(x) \cdot f(y)}{f(x+y)} = 1 + [(1 - \{f(x)\}^2)(1 - \{f(y)\}^2)]^{\frac{1}{2}}.$$

6. What must be added to each of the numbers 8, 56, 200 so as to give three numbers in continued proportion ?

7. If $a = \sqrt{\frac{1-e}{1+e}}$, find in as simple a form as possible the value of $\frac{1-a}{1+a}$.

128 TEST PAPERS IN ELEMENTARY ALGEBRA

8. I have 30 miles to go to a railway station and $4\frac{1}{2}$ hours to get there in. On my bicycle I go at the rate of 15 miles an hour, but my front tyre is likely to burst, in which case I shall have to walk with the machine at 3 miles an hour. How many miles must I ride before I am free from anxiety as to whether I shall catch the train?

127.

1. If $a = x^k$, $b = x^l$, $c = x^m$, express as a power of x the fraction $\frac{a^x b^y}{c^r}$.

2. Solve the equations:

$$(1) \frac{2x^2 - 1}{5} - \frac{3x + 1}{4} = \frac{9}{10}.$$

$$(2) c^2x + 2a^2y = (c + a)(cx + 2ay) = (c - a)^2.$$

$$(3) \sqrt{x+3} - \sqrt{x-2} = 1.$$

3. The first and second terms of an arithmetic progression are $1 - x$ and $x - 2$; if the n th term is $5n - 8$, find the value of x .

4. If t is proportional to $p \cdot v^{\frac{3}{2}}$, and if $t = 224$ when $p = 28$, $v = 16$, what is the value of t when $p = 32$, $v = 25$?

5. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$, prove that

$$(a + b + c + d)^2 = (a + b)^2 + (c + d)^2 + 2(b + c)^2.$$

6. Find the gradients of the functions $\frac{1}{2}(x + 3)$ and $3 - x^2$ when $x = 1$. Draw the graphs of these functions, and interpret geometrically the fact that the product of the gradients, when $x = 1$, is -1 .

7. If $(1 + x^2)f(x) + (1 + x^2 + x^3)\phi(x) \equiv 1$ and if $f(x)$, $\phi(x)$ are rational integral functions of degree 2 and 1 respectively, find $f(x)$ and $\phi(x)$.

8. From a cask of wine, 9 gallons are drawn and replaced by water; 12 gallons of the mixture are now drawn and replaced by water. If the amounts of wine and water now in the cask are equal, find the capacity of the cask.

R.B.A.N. M'S HIGH SCHOOL,

128.

Bangalore

1. Simplify

$$(1) (2x^2 - 19x + 35)(2x^2 - 13x + 15) / (4x^2 - 16x + 15).$$

$$(2) \left(\frac{1}{x+y-z} - \frac{1}{x+y+z} \right) \div \left(\frac{1}{(x+y)^2 - z^2} - \frac{1}{(x+y)^2 + z^2} \right).$$

2. If the sum of the first n terms of a series is $7n^2 - 5n$ for all values of n , prove that it is an arithmetical progression, and find the common difference.

3. If $(x-a)(y-a) = \beta^2$, prove that

$$\frac{1}{x-a-\beta} + \frac{1}{y-a-\beta} + \frac{1}{\beta} = 0,$$

and that

$$\frac{1}{x-a+\beta} + \frac{1}{y-a+\beta} - \frac{1}{\beta} = 0.$$

4. Solve the equations:

$$(1) \left(\frac{x-1}{5} \right)^2 = 8 - \frac{x+1}{3}.$$

$$(2) ax + by = a^2 + 2ab - b^2, \quad bx + ay = a^2 + b^2.$$

5. (1) If $\frac{a}{b} = \frac{c}{d}$, prove that

$$\frac{a^{2n} + b^{2n} + c^{2n} + d^{2n}}{a^{-2n} + b^{-2n} + c^{-2n} + d^{-2n}} = (abcd)^n.$$

$$(2) \text{ Simplify } \frac{2\sqrt{3} - \sqrt{11}}{(2\sqrt{3} + \sqrt{11})^2} + \frac{2\sqrt{3} + \sqrt{11}}{(2\sqrt{3} - \sqrt{11})^2}.$$

6. If $z = x\sqrt{1+y^2} + y\sqrt{1+x^2}$, prove that

$$\sqrt{1+z^2} = xy + \sqrt{(1+x^2)(1+y^2)}.$$

130 TEST PAPERS IN ELEMENTARY ALGEBRA

7. The number of competitors in a race varies directly as the value of the prize and inversely as the square of the length of the course. There are 25 entries for a half-mile race, the prize for which is 5£. How many will enter for a mile race, the prize being worth 8£?

8. Men standing four deep form a hollow square. If they stand three deep, they can form a hollow square with ten more men in a row. Find the number of men.

129.

1. If $a = 4\sqrt{2}$, $b = 3\sqrt{5}$, $c = \sqrt{5}$, find the value of

$$\frac{(a+b+c)(b+c-a)}{(c+a-b)(a+b-c)}.$$

2. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$, prove that $\left(\frac{ab+bc+ca}{bc+cd+db}\right)^3 = \frac{a^2}{d^2}$.

3. Solve the equations :

$$(1) \frac{x-1}{x+1} + \frac{x-3}{x+3} = \frac{2(x-2)}{x+2}.$$

$$(2) \frac{x}{x+b-a} + \frac{b}{x+b-c} = 1.$$

4. The sum of ten terms of an arithmetical progression is 145 and the sum of its 4th and 9th terms is five times the 3rd term. Find the series.

5. Find the value of c if $12x^2 + xy - 6y^2 - 29x + 8y + c$ can be expressed as the product of two linear functions of x and y .

6. The perimeter of a right-angled triangle is six times the shortest side. Find the ratio of the two sides containing the right angle.

7. If x and y are positive quantities such that x is less than $\frac{1-y}{1+y}$, prove that y is less than $\frac{1-x}{1+x}$.

8. A starts from P to Q half an hour after B , overtakes him midway between P and Q and arrives at Q at 2 p.m.; after waiting $7\frac{1}{2}$ minutes at Q , he returns and meets B ten minutes later. At what time did A leave P ?

130.

1. If $x - y = 2^2$ and $x^2 - y^2 = 8^2$, prove that $x^3 - y^3 = 28^2$.

2. Factorise $(1-a)^2b^4 - 2(1+a^2)b^2 + (1+a)^2$, and prove that

$$a(b+c-a)^2 + b(c+a-b)^2 + c(a+b-c)^2 \\ + (b+c-a)(c+a-b)(a+b-c) = 4abc.$$

3. If $\frac{a}{c} = \frac{c}{b}$, prove that

$$(a^2 + b^2 + 2c^2)(a + b + 2c)^2 = [(a + c)^2 + (b + c)^2]^2.$$

4. Simplify $\frac{\sqrt{12}}{(1 + \sqrt{2})(\sqrt{6} - \sqrt{3})}$.

5. The volume of a sphere varies as the cube of its radius. If a sphere of radius x inches is made into a hollow sphere whose internal radius is y inches, find its thickness.

6. The angles of a pentagon are in arithmetical progression, and the greatest is twice the least. Find the least angle.

7. Prove that the gradient at the point $(6, 12)$ on the graph of $y = \frac{1}{3}x^2$ is the same as the gradient at any point of the graph of $2y = 8x + 3$.

8. A person walks 20 times round a square field and finds he has gone as many miles as the field contains acres. What is the area of the field?

PAPERS 131-140 include

Simultaneous quadratics; arithmetical and geometrical progressions.

131.

1. If $a = x^2 - yz$, $b = y^2 - zx$, $c = z^2 - xy$, prove that

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(x + y + z)(ax + by + cz).$$
2. Simplify (1) $\left(\frac{1-x}{x} + \frac{4}{1-x}\right) \div \left(\frac{1+x^2}{x^2} - \frac{4}{1+x^2}\right)$.
 (2) $(\sqrt{37} + 6)^{\frac{1}{2}}(\sqrt{37} - 6)^{-\frac{1}{2}}.$
3. Solve the equations :
 (1) $(x+3)(x+2)(x+5)$

$$= (x-3)(16x+5) + (x+3)(x-4)(x-5).$$

 (2) $x + y = z = 2xy$; $\frac{1}{x} - \frac{1}{y} = 1.$
 (3) $x^2 - 2xy - y^2 = 2$; $xy + 2y^2 = 5.$
4. Two numbers are in the ratio of 5 : 4. If 8 is taken from each, the remainders are in the ratio of 3 : 2. Find the numbers.
5. (1) If a, b, c, d are in geometrical progression, prove that $(ab + bc + cd)^2(a - c)(b - d) = (a^3 - c^3)(b^3 - d^3).$
 (2) Find the sum of the numbers in the n th bracket in the following : [1]; [2 + 4]; [8 + 16 + 32]; ...
6. A motorist A starts from X and drives to Y , which is 15 miles away ; 15 minutes later a cyclist B leaves Y and rides to X , taking 65 minutes for the journey ; A returns to X without waiting at Y , and arrives there at the same time as

B. Where did *A* first meet *B*? Give both a graphical and an algebraic solution.

7. What value of x makes $x^4 + 6x^3 + 11x^2 + 3x + 31$ a perfect square?

8. Prove that values of x, y, z can be found which satisfy each of the four equations:

$$7x - 13y + 6z + 13 = 0.$$

$$5x + 4y + 7z + 4 = 0.$$

$$8x - y + z + 5 = 0.$$

$$9x - y + 4z + 7 = 0.$$

132.

1. Prove that $\frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} + \frac{(x-a)(x-b)}{(c-a)(c-b)}$ is equal to 1.

2. The first and last of 46 terms in arithmetical progression are -7 and $+62$. Find their sum and also the two middle terms.

3. Solve

$$(1) \ 4\{x - 3[x - 2(x - 1)]\} = 24.$$

$$(2) \ x^2 + 2xy = 3; \quad y^2 - xy = 4.$$

$$(3) \ x + y + z = 19; \quad x^2 + y^2 + z^2 = 133; \quad yz = x^2.$$

4. *A* takes x hours longer than *B* to walk y miles, but if he doubles his pace he takes z hours less time than *B*. Find *A*'s pace in terms of x, y, z .

5. (1) If the sum of n terms of an arithmetical progression whose first two terms are a and b is $ma + kb$ for all values of a, b , prove that $(m+k)^2 = m + 3k$.

(2) If a, b, c, d are in geometrical progression, prove that $\frac{ab + bc + ca}{bc + cd + db} = \frac{a^{\frac{2}{3}}}{d^{\frac{2}{3}}}$.

6. Two companies in a regiment appeared in the field in the strengths $9:11$. At the end of the day the relative strength

was 5:6. If 35 per cent. of the men in the two companies were put out of action, and of these 34 men belonged to the first company, find the number in each company at first.

7. If $a = \frac{1}{2}(-1 + \sqrt{-3})$ and if $a + b + 1 = 0$, find the values of a^3 ; b^3 ; $a^{3n} + b^{3n}$; $a^{3n+1} + b^{3n+1}$, where n is any positive integer.

8. If the minute hand of a clock moves in the opposite direction to the hour hand, find when the hands of a clock are coincident between three and four o'clock.

133.

1. If $x = a + b + \frac{(a-b)^2}{4(a+b)}$ and $y = \frac{a+b}{4} + \frac{ab}{a+b}$, prove that
 $(x-a)^2 - (y-b)^2 = b^2$.

2. Solve the equations:

$$(1) \frac{x-y}{y-1} = \frac{y+1}{x} = \frac{5}{8}.$$

$$(2) \sqrt{x+4} + \sqrt{x-3} = \sqrt{7}.$$

$$(3) xy + y^2 = 4, \quad x^2 - xy + 2y^2 = 8.$$

3. If £ x . $ys.$ is six times £ y . $xs.$, find the values of x and y , each sum of money being expressed in its simplest form.

4. If $a^2 + d^2 = b^2 + c^2$, prove that

$$\frac{(b+c+d-a)(a+b+c-d)}{(c+d+a-b)(d+a+b-c)} = 1.$$

5. What is the smallest number of terms which may be taken of the geometric progression 4, 6, 9, ... to give a sum exceeding 8000?

6. Prove that the remainder when $x^4 + 7x + 3$ is divided by $x-1$ is the same as the remainder when $x^4 - 6x + 7$ is divided by $x-2$.

7. Divide 1 into two parts such that the sum of their cubes is $\frac{1}{3}$.

8. Two casks contain mixtures of wine and water in the ratio 8:3 and 5:1 respectively. In what ratio must liquid be drawn from each cask to give a mixture in the ratio 4:1?

134.

1. If $x = \frac{ab - cd}{a - b - c + d}$, prove that $(x + a)(x - b) = (x + c)(x - d)$.

2. Solve the equations :

$$(1) \frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{4}{x-2}.$$

$$(2) 2^{2x+2} + 2^{2-2x} = 17.$$

3. Simplify

$$(1) \left(\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} \right)^{\frac{1}{2}}.$$

$$(2) \frac{x+2 - (x+2)^{-1}}{x+2 - 4(x+5)^{-1}} \times \frac{x+4 - 4(x+4)^{-1}}{x+4 - (x+4)^{-1}}.$$

4. If a, x, y, b are in arithmetical progression and c^3, x, y, d^3 in geometric progression, prove that $a + b = cd(c + d)$.

5. Eliminate t from the equations :

$$x = 3\left(t + \frac{1}{t}\right), \quad y = 2\left(t^2 + \frac{1}{t^2}\right).$$

6. A horse which cost $x\text{£}$ is sold for $\text{£}24$ at a loss of x per cent. Find x .

7. If $\frac{v^2}{r} = \frac{g}{289}$, calculate v given that $r = 4000$, $g = \frac{32 \cdot 2}{5280}$,

and prove that $\frac{2\pi r}{v \times 60 \times 60}$ is approximately 24, where $\pi = 3 \cdot 142$.

136 TEST PAPERS IN ELEMENTARY ALGEBRA

8. A cistern with a leak in the bottom can be filled by one pipe in 20 minutes and by two pipes in 8 minutes. In how many minutes can it be filled by three pipes, all the pipes being the same size?

135.

1. If $\frac{x}{y+z} = a$, $\frac{y}{z+x} = b$, $\frac{z}{x+y} = c$, prove that

$$\frac{x^2}{a-abc} = \frac{y^2}{b-abc} = \frac{z^2}{c-abc}.$$

2. Find the square root of

$$\left[\left(\frac{1}{a} + \frac{1}{b} \right)^2 - \frac{1}{ab} - \frac{4}{(a-b)^2} \right] \div \left[\left(\frac{1}{a} - \frac{1}{b} \right)^2 - \frac{1}{ab} \right].$$

3. If

$3x^2 - 7x - 1 \equiv a(x-1)(x-2) + b(x-2)(x-3) + c(x-3)(x-1)$
for all values of x , find the values of a , b , c .

4. If the sum of the first p terms of an arithmetical progression is zero, prove that the sum of the next q terms is $-\frac{a(p+q)q}{p-1}$, where a is the first term.

5. Prove that $\log \frac{7}{16} - 2 \log \frac{5}{9} + \log \frac{3}{2} \frac{2}{43} = \log 2$.

6. A square of carpet is cut up into strips so as to cover a border $a+b$ feet wide round the floor of a room whose length is $10a$ feet and breadth $10b$ feet. What was the length of the side of the square?

7. If a, b, c, d, e are consecutive integers, prove that 5 is a factor of $a^2 + b^2 + c^2 + d^2 + e^2$.

8. A fox and an eagle lived at the top of a cliff of height h , whose base was distant mh from a farm. The fox descended

the cliff and went straight to the farm; the eagle flew up to a height x , and then flew straight to the farm. If the distance traversed by each was the same, find x in terms of m, h .

136.

1. Simplify

$$(1) \left\{ \left(\frac{a+b}{a} \right)^2 - \left(\frac{a-b}{b} \right)^2 \right\} \div \left(1 - \frac{2ab}{a^2 - b^2} \right).$$

$$(2) \frac{1}{y \left(\frac{x}{y} - \frac{y}{x} \right)} - \frac{1}{x \left(\frac{x}{y} + \frac{y}{x} \right)} + \frac{1}{\left(\frac{1}{x} + \frac{1}{y} \right) (x^2 + y^2)}.$$

2. If $\frac{a}{xy} = \frac{b}{x^2} = \frac{c}{yz}$, prove that $\frac{x}{ab} = \frac{y}{a^2} = \frac{z}{bc}$.

3. If a, b are consecutive integers, prove that $a^2 + b^2 + a^2b^2$ is a perfect square.

4. Solve the equations (1) $2xy + y^2 = 16, 2x^2 - xy = 12$.

$$(2) x(y+1) = y(a+1) = a(x+1).$$

5. During any year the excess of births over deaths causes an increase of h per cent. of the population, and at the end of the year a fixed number A of the population emigrate. If the population at the beginning of a certain year is P , how much is it at the end of n years from that time?

6. Find the sum of all numbers less than 1000 which are not divisible by either 2 or 5.

7. Two cyclists travel, one from A to B , the other from B to A , by the same road at uniform speeds. They start at the same time. One reaches B $2\frac{1}{2}$ hours and the other $3\frac{3}{5}$ hours after they meet. How long does each take over the journey?

138 TEST PAPERS IN ELEMENTARY ALGEBRA

8. A string of length l has its ends tied to the two ends of a heavy straight stick of length a , and is slung over a small smooth peg, so that the string and stick are vertical. Find the height of the peg above the mid point of the stick.

137.

1. If $a^2 = b^2 + c^2 - 2bc(2x^2 - 1)$, prove that

$$x = \left[\frac{(a+b+c)(b+c-a)}{4bc} \right]^{\frac{1}{2}}.$$

2. Find the value of $\frac{x + \frac{a}{b}}{x + \frac{a}{a}} - \frac{x - \frac{a}{b}}{x - \frac{a}{a}}$ when $x=1$, and find the

other value of x for which this expression has the same value.

3. Factorise $(xy - ab)^2 - (bx - ay)^2$, and prove that

$$a(b-c)(b+c-a)^4 + b(c-a)(c+a-b)^4 + c(a-b)(a+b-c)^4 \\ = kabc(a-b)(b-c)(c-a),$$

where k is a constant, and find the value of k .

4. If a, b, c, d are in geometrical progression, and if $b - c = 2$ and $a - d = 7$, find the value of a .

5. In a series of right-angled triangles, one side has in succession the lengths 1×4 , 2×6 , 3×8 , 4×10 , ... inches, and the hypotenuse exceeds this side by one inch. Prove that the lengths of the other side form an arithmetical progression.

6. If $\frac{a}{b} : \frac{b}{c} = \frac{c}{d} : \frac{d}{e}$, prove that $\frac{b^4 + a^2c^2}{b^4 - a^2c^2} = \frac{d^4 + c^2e^2}{d^4 - c^2e^2}$, and if $\frac{a}{b} = \frac{b}{d}$, prove that $d = e$.

7. A grocer has teas worth 2s. 8d., 1s. 8d., 1s. 4d. per lb. How must he mix them for a customer who wants a 56 lb.

chest at 1s. 10d. per lb., in which there shall be 2 lb. more of the best than of the worst tea?

8. If $f(x) = \frac{1}{1-x}$, prove that $f[f\{f[f(x)]\}] = f(x)$.

138.

1. If $x + y = xy = 1$, prove that $x^3 = y^3 = -1$.

2. Prove that the equations

$$x^3 + x^2 + x + 6 = 0; \quad x^4 - x^3 + x^2 + 2x = 6$$

have two roots in common, and find the other roots.

3. Solve the equations (1) $9y + x = x(x + y); \quad x - y = 2$.

(2) $x^2 + y^2 = c^2, \quad (ax - by)^2 = c^2(a^2 + b^2)$.

4. The p th term of an arithmetical progression is u , and the q th term is v ; find the r th term.

5. If s is the sum to infinity of the geometrical progression whose first two terms are u, v , prove that the sum of n terms of an arithmetical progression, whose first two terms are u, v , is $nu - \frac{n(n-1)}{2} \cdot \frac{u^2}{s}$.

6. The pressure of wind on a plane surface varies jointly as the area of the surface and the square of the wind's velocity. If the pressure on a square foot is 1 lb. when the wind is moving at 15 miles an hour, find the velocity of the wind when the pressure on a square yard is 16 lb.

7. Two numbers differ by 6; prove that if 9 be added to the product, the sum will be a perfect square. If the square root of this be 13, find the two numbers.

8. A person gives to each of five boys a certain number of pence which is represented by two digits; and the whole sum expressed in shillings and pence is represented by the same two digits. How much did he give each boy?

139.

1. If $ab + bc + ca = 0$, prove that $\frac{a}{b} + \frac{a}{c} = \frac{b}{c} + \frac{b}{a} = \frac{c}{a} + \frac{c}{b}$.

2. Solve the equations

$$(1) \quad x^2 + xy = 90, \quad xy + y^2 = 10.$$

$$(2) \quad (x-1)\sqrt{4-x} = 2.$$

Determine by inspection one root of the equation

$$(c+a-2b)x^2 + (a+b-2c)x + (b+c-2a) = 0,$$

and then write down the other root.

3. Simplify (1) $\frac{a^3 - b^3}{a^2b + ab^2} \left(\frac{1}{a-b} - \frac{1}{a+b} \right) \div \frac{1}{a^2 + ab}$.
 (2) $9^{2n+1} \times 6^{2n-3} \div (3^{5n-2} \times 18 \times 4^{n-2})$.

4. Prove that $\frac{101 + 103 + 105 + \dots + 199}{1 + 3 + 5 + \dots + 99} = 3$.

5. If p, q, r, s are four consecutive terms of the series whose n th term is $3^n + 2n$, prove that $3p + 5r = 7q + s$.

6. If $\frac{a}{b} = \frac{b}{c}$, prove that (1) $b^2(a^2 - b^2) = a^2(b^2 - c^2)$.

$$(2) \quad \frac{a-b}{b-c} = \sqrt{\frac{a}{c}}.$$

7. If a solid sphere, 3 inches in diameter, weighs 30 ounces, what is the weight of a solid sphere of the same material, whose diameter is 5 inches?

8. A bill before Parliament is lost on a division, there being 600 votes recorded. Afterwards it was carried by the same voters by twice as many votes as it was before lost by, and the new number in its favour was to the original number against it as 5 : 4. How many members changed their minds?

140.

1. If $y - z = ax$, $z - x = by$, $x - y = cz$, prove that

$$abc + a + b + c = 0.$$

2. If the sum of n terms of the series

$$1 \cdot 3 + 3 \cdot 3^2 + 5 \cdot 3^3 + 7 \cdot 3^4 + 9 \cdot 3^5 + \dots$$

is $(an + b)3^n + c$ for all values of n , find the values of a , b , c .

3. Factorise (1) $1 + y(1 + x)^2(1 + xy)$.

(2) $x^3 - 2x^2 - 29x + 30$.

4. Solve the equations :

(1) $x^2 + 3y^2 = 43$, $x^2 + xy = 28$.

(2) $\sqrt{x+a} = \sqrt{x} + \sqrt{b}$.

5. Find the area of the figure whose vertices taken in order are $(0, 4)$, $(2, 7)$, $(7, 10)$, $(9, 9)$.

6. Find the three smallest positive integers which are such that their squares are in arithmetical progression, and the difference between the two smaller numbers is double the difference between the two larger.

7. If $ABCDE$ are five points on a straight line, and if C is the mid point of BD , prove that $BC^2 + CE^2 - DE^2 = BD \cdot CE$.

8. Two rectangular lawns have the same area, a^2 sq. feet, but the perimeter of one is one-fourth larger than the perimeter of the other, which is a square. Find its dimensions.

142 TEST PAPERS IN ELEMENTARY ALGEBRA

PAPERS 141-150 include

Elementary theory of quadratics; harmonical progressions.

141.

1. If $x + y = 1$, prove that $x^3(y + 1) - y^3(x + 1) = x - y$.
2. If $f(x) = \frac{a^x - a^{-x}}{a^x + a^{-x}}$, prove that $f(x + y) = \frac{f(x) + f(y)}{1 + f(x) \cdot f(y)}$, and express $f(3x)$ in terms of $f(x)$.
3. (1) If a^2, b^2, c^2 are in arithmetical progression, prove that $b + c, c + a, a + b$ are in harmonical progression.
(2) If the n th term of a series is $2^{n-1} + 2n$, find the sum of n terms.
4. Solve the equations:
 - (1) $x^2 - y^2 = 11$; $x^2 - z^2 = yz = 20$.
 - (2) $\frac{x-a}{b} + \frac{x-b}{a} = \frac{b}{x-a} + \frac{a}{x-b}$.
5. (1) Find the value of a if the equation $x^2 - (a + 2)x + 2a + 1 = 0$ has equal roots.
(2) If $a + b = -2$ and $ab = -5$, find the value of $a^2 + b^2$.
6. A train started for a journey of 60 miles, which was timed to take 90 minutes; but on the way it ran into a fog, and the driver was compelled to slacken speed, so that it arrived 25 minutes late. Had the fog come on 6 miles farther from the place of destination, the train would have been 40 minutes late. Find the rate of travelling in the fog.
7. Along the straight line AB the following distances are

measured in order: $AB = \log pq$, $BC = \log qr$, $CD = \log rt$, $DE = \log tp$. If F is the mid point of AE , prove that

$$AF = \log pqrt.$$

8. If $x^2 + ax + b$ and $bx^2 + ax + 1$ are unequal but have a common factor, prove that $1 + b = \pm a$.

142.

1. If $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 1$, prove that $y^2 - 2y(1 + 3x) + (1 - x)^3 = 0$.

2. (1) If $\frac{a}{b} = \frac{c}{d}$, prove that

$$(a + b + c + d)(a - b - c + d) = (a + b - c - d)(a - b + c - d).$$

(2) What value of x makes $3 - x$, $6 - x$, $7 + x$, $19 + x$ proportionals?

3. (1) If $a_1, a_2, \dots, a_n$ are in harmonical progression, prove that $a_1a_2 + a_2a_3 + \dots + a_{n-1}a_n = (n - 1)a_1a_n$.

(2) If the sum of n terms of a series is $3n^2 - 5n$, prove that the series is an arithmetical progression, and find the 10th term.

4. Find the factors of

$$(1) y^2z^2(y - z) + z^2x^2(z - x) + x^2y^2(x - y).$$

$$(2) a^3 - 3a^2b + 3ab^2 + 7b^3.$$

5. A specimen of bronze consists of 85 per cent. copper, 10 per cent. zinc, and 5 per cent. tin, and a specimen of bell metal consists of copper and tin only; the two are fused together, and the mixture is found to contain 84 per cent. copper, 8 per cent. zinc, and 8 per cent. tin. Find the proportion of copper and tin in the bell metal.

6. (1) One of the roots of the equation $x^2 - 2x + k = 0$ is $\frac{5}{4}$; what is the other?

(2) Find the condition that one root of the equation $(a - 1)x^2 - 2(a - 1)x + a - 2 = 0$ is double the other.

144 TEST PAPERS IN ELEMENTARY ALGEBRA

7. The velocity of discharge (v) in feet per second of water through a long pipe is given by the equation $v^{1.87} = \frac{H \cdot d^{1.4}}{.0004L}$, where H is the head, L the length of the pipe, and d its diameter, all expressed in feet. Find the velocity of discharge through a pipe one mile in length and 18 inches in diameter, if there is a head of 35 feet.

8. Find a function of x and y of the first degree which vanishes when $x=1$, $y=2$, and also when $x=2$, $y=-1$.

143.

1. Prove that $a+b+c$ is a factor of

$$(ax+by+cz)^2+bc(y-z)^2+ca(z-x)^2+ab(x-y)^2.$$

2. Factorise $2x^2+3xy+y^2-6x-4y+4$, and solve the equations $2x^2+3xy+y^2-6x-4y+4=2x^2-xy-y^2-4x+y+2=0$.

3. If a , b , c are in arithmetical progression and b , c , d in harmonical progression, prove that $\frac{a}{b} = \frac{c}{d}$.

4. Find the value of a if one root of the equation $x^2-(a+3)x+2(a+1)=0$ is three times the other.

5. What number must be added to each of the numbers 1, 17, 65 to give three numbers in geometrical progression?

6. Solve the equations:

$$(1) \frac{3x-16}{x-3} + \frac{2x+3}{x-2} = \frac{5x-2}{x+1}.$$

$$(2) x^2+y^2=13, xy-2x-2y+4=0.$$

7. A man walks a certain distance and begins to return at double the pace, but gets tired, and reduces his speed to one-third of his original pace. If the times taken in going and

returning are equal, compare the times during which he walked at the various paces.

8. The n th term of the set of numbers 7, 20, 57, 166, ... is $a \cdot 3^n + bn$; find the values of a , b . Hence find the sum of the first n numbers.

R.S.A.N. M'S HIGH SCHOOL

144.

Bangalore.

1. Simplify (1) $\frac{x(x+1) - y(y-1)}{x(x-1) - y(y+1)}$.

(2) $\left[\left(\frac{1+x}{1-x} \right)^2 - \left(\frac{1-x}{1+x} \right)^2 \right] \div \left[\frac{1+x}{1-x} + \frac{1-x}{1+x} \right]$.

2. Solve the equations :

(1) $(x-2)^3 + x^3 = 2(x-1)^3 + 12$.

(2) $2^{x+y} = 8, 3^{x-y} = 1$.

3. Find the sum of the squares of the roots of the equation

$$2x^2 - x = 7.$$

Prove also that the roots of this equation are half the roots of the equation $x^2 - x = 14$.

4. Prove that the difference between the arithmetic mean of the squares of $a+x$ and $b+x$ and the square of the arithmetic mean of $a+x$ and $b+x$ is independent of x .

5. If the m th term of a harmonical progression is n , and if the n th term is m , prove that the mn th term is 1.

6. Find correct to three significant figures the value of $\pi wh(x^2 - y^2)$, where $\pi = 3.142$, $w = 2.17$, $h = 6.351$, $x = 4.072$, $y = 3.843$.

7. If $\frac{a}{b} = \frac{b}{c}$, prove that $\frac{a+b+c}{a-b+c} = \frac{(a+b+c)^2}{a^2+b^2+c^2}$.

D. T. P.

K

8. A man spends on charitable objects an annual amount proportional to the square of his income. He spends £3 more when his income is £1200 than when it is £900. Find his charitable expenditure in each case.

145.

1. (1) If $x = 1 + \sqrt{2}$, prove that $x^4 + 1 = 6x^2$.
 (2) If $x = 3\sqrt{2} - 2\sqrt{3}$ and $y = 3\sqrt{2} + 2\sqrt{3}$, prove that $x^2 + y^2 = 10xy$.
2. (1) If a, b, c are in arithmetical progression, prove that $2a - 3b, 2b - 3c, 3b - 4c$ also form an arithmetical progression.
 (2) Prove that the sum of any number of terms of the series $1 + 3 + 5 + 7 + \dots$ is a perfect square.
3. If α, β are the roots of the equation $x^2 - 3x - 11 = 0$, find the equation whose roots are (1) $\alpha + 2, \beta + 2$, and (2) $2\alpha, 2\beta$.
4. A has $p\text{£}$; B then pays him $\frac{1}{n}$ of what he (B) has, and finds that he now has $\frac{1}{m}$ of what A has. How much had B at first?
5. Determine by inspection one root of the equation $(x + b + c)(x + c + a)(x + a + b) + abc = 0$.
6. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, prove that $(a^3 + c^2e)(b + d + f)^3 = (b^3 + d^2f)(a + c + e)^3$.
7. (1) Find the gradient of the function $3t + t^2$ when $t = 2$.
 (2) A body moves so that it travels $3t + t^2$ feet in t seconds for all values of t . What is its average speed for the interval from $t = 2$ to $t = 2 + h$ seconds, and what is its velocity at the end of 2 seconds?

8. A man bicycles 40 miles, partly at 15 miles an hour and partly at 12 miles an hour. For how long must he ride at the former rate in order to complete the whole distance in 3 hours?

146.

1. (1) Prove that

$$(a^2 + b^2 + c^2 + d^2)(p^2 + q^2 + r^2 + s^2) = (ap + bq + cr + ds)^2 + (aq - bp + cs - dr)^2 + (ar - bs - cp + dq)^2 + (as + br - cq - dp)^2.$$

(2) If each of the numbers x, y, z can be expressed as the sum of four squares, prove that xyz can also be expressed as the sum of four squares.

2. If the sum of the roots of the equation

$$x^2 - (a + 3)x + 2(a + 1) = 0$$

is two-thirds of their product, find the value of a .

3. If a, b, c, d are in geometrical progression, prove that

$$a^2 - c^2 = (a + c)(a + d) - (b + c)(b + d).$$

4. (1) Form the equation whose roots are $0, 1, 2 + \sqrt{3}, 2 - \sqrt{3}$.

(2) Find the relation between p, q if the roots of the equation $x^2 + px + q = 0$ differ by $3p$.

5. Solve the equations :

$$(1) (b - a)x - (c - b)y = c - a, (a - c)x - (b - a)y = b - c.$$

$$(2) \frac{x + y}{3} = \frac{z}{2} = \frac{3y + 4z}{13} = \frac{5z + 5x + 4}{18}.$$

6. Men, standing four deep, form a hollow square. Standing three deep, they could form a hollow square with two more men in the front rank than before. Find the number of men.

7. If n harmonic means are inserted between a and b prove that the r th of these is $(n+1)\left(\frac{r}{b} + \frac{n-r+1}{a}\right)^{-1}$.

8. The sum of n terms of a series is $5n^2 - 4n$. Prove that it is an arithmetical progression, and that the 90th term is eleven times the 9th term.

147.

1. Subtract $3 \cdot 2^n$ from 2^{n+3} , and multiply the result by $6 \cdot 2^n$. What is the least integer by which the final answer must be multiplied in order that the result may be (1) a perfect square, (2) a perfect cube?

2. (1) Find the relation between p and q if the ratio of the roots of the equation $x^2 + px + q = 0$ is $2:3$.

(2) Find the sum of the cubes of the roots of the equation $x^2 - x = 1$.

3. Eliminate t from the equations $x = 2t - \frac{3}{t}$, $y = t + \frac{1}{t}$.

4. Find the value of n if the sum of n terms of the series $2 + 5 + 8 + 11 + \dots$ is equal to the sum of n terms of the series $47 + 45 + 43 + \dots$.

5. Simplify $\frac{1}{\sqrt{5} + \sqrt{3}} + \frac{2}{3 + \sqrt{5}} - \frac{3}{3 + \sqrt{3}}$.

6. The hypotenuse of a right-angled triangle is less than the sum of the other two sides by 8 feet, and the area equals 180 sq. feet. Find all the sides.

7. If $ax - by = 1$ and $ax^2 - by^2 = \frac{1}{a-b}$, find the values of x , y , and hence prove that $ax^n - by^n = (a-b)^{1-n}$.

8. Two casks, each containing 20 gallons, are filled, one with water and the other with spirit. x gallons are drawn from each cask, mixed, and the casks are again filled up with the mixture; when this is done a second time, the quantity of spirit in the first cask is to that in the second as 3 : 5. Find the value of x .

148.

1. Simplify
$$\frac{(x - x^{-1})(x^2 + x^{-2}) + (x + x^{-1})(x^2 - x^{-2})}{(x + x^{-1})^2 + x^2 + x^{-2}}.$$

2. Prove that $(x - 1)^2$ is a factor of $x^{10} - 10x + 9$.

3. Prove that the difference of the roots of $x^2 - 3x - 5 = 0$ is equal to that of the roots of $x^2 - 5x - 1 = 0$.

4. If x, y, z are in geometrical progression and if x, a, y and y, b, z are in arithmetical progression, prove that

$$\frac{x}{a} + \frac{z}{b} = 2.$$

5. Solve the equations

(1) $x^2 - 2(ab - c)x = a^2(a^2 - 2ab + 2c).$

(2) $\frac{x^2 + (y - 1)^2}{4} = \frac{y^2 + (x + 1)^2}{9} = y - 1.$

6. The perimeter of a rectangle is 12 inches and its area is 6 sq. inches. If the lengths of its sides, measured in inches, are the roots of the equation $x^2 + bx + c = 0$, find the values of b and c .

7. If $f(x) \equiv 2^x - x$, prove that

$$f(x + 2) - 3f(x + 1) + 2f(x) = f(0).$$

8. Find the price of butter per pound when a quarter of a pound more in three shillings' worth lowers the price two-pence per lb.

149.

1. The horse-power required to drive a certain ship is estimated from experiment on a model as follows :

Horse-power = $H = 290$	560	1044	1810	2300
Speed in knots = $V = 5$	7	9	11	12

Prove that these figures agree approximately with a law of the type $H = a + bV^3$, where a, b are constants, and find the values of a, b .

Calculate the horse-power necessary for a speed of 18 knots.

2. Solve the equations :

$$(1) \frac{5}{4+y} - \frac{9}{20-x} = 9y - 5x - 26 = 0.$$

$$(2) \frac{ax}{b} - \frac{b}{ax} = b - \frac{1}{b}.$$

3. If $a, b, c, d, \dots, k, l$ are in harmonical progression, prove that al is the average of the quantities $ab, bc, cd, \dots, kl$.

4. Prove that $x - 2$ and $x + 3$ are factors of

$$x^3 - x^2(2a - 1) - 2x(a + 3) + 12a$$

for all values of a , and find the value of a if $x + 2$ is also a factor.

5. The sum of n terms of the series

$$-3 + 9 + 33 + 69 + 117 + \dots$$

is of the form $an^3 + bn$, where a and b are constants. Find the values of a, b ; hence find (1) the sixth term, (2) the n th term of the series.

6. Find a if the roots of the equation

$$(a + 1)x^2 - 2(a + 3)x + 4a - 3 = 0$$

are equal.

7. Find the greatest possible number of points of intersection of n straight lines and m circles.

8. If a tap can fill a spherical vessel, radius a feet, in x minutes, how long will n similar taps take to fill a spherical vessel, radius b feet?

150.

1. Find correct to three significant figures the values of
(1) $(3.25)^{-1.7}$; (2) $\frac{a(1-r^n)}{1-r}$, where $a = 3.62$, $r = .835$, $n = 10$.

2. If a, b, c, d are in geometrical progression, prove that
 $(a+b+c+d)^2 - (a+b-c-d)^2 = 2[(a+b)(c+d) + (b+c)^2]$.

3. Find the positive integral value of x which makes
 $3x^2 - 17x + 11$ nearest to 1000.

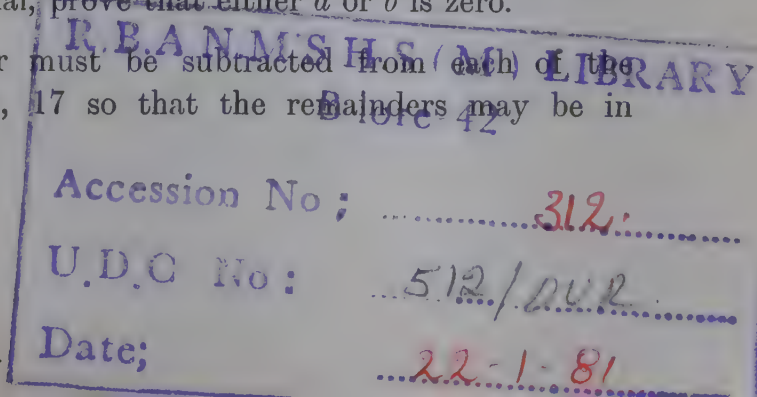
4. Find the sum of all numbers of three different digits not beginning with a zero.

5. Prove that the roots of the equation $4x^2 - 37x + 49 = 0$ are the squares of the roots of the equation $2x^2 - 3x - 7 = 0$.

6. Two trains are proceeding in the same direction at uniform rates. The quicker train, which is in front, gains $\frac{1}{n}$ of a mile every minute, and also $\frac{1}{n}$ of a minute every mile on the slower train. Determine the speed of the faster train, and prove that, if n is positive, this cannot be less than 60 miles an hour.

7. If $x^2 + bx - a^2$ and $x^2 + ax - b^2$ have a common factor, and if a and b are unequal, prove that either a or b is zero.

8. What number must be subtracted from each of the numbers 8, 10, 13, 17 so that the remainders may be in proportion?



PAPERS 151-160 include

Permutations and combinations.

151.

1. Find the value of $\frac{px+q}{qx+p} - \frac{px-q}{qx-p}$ when $x=1$, and find another value of x for which this function has the same value.

2. In x years' time a man will be twice as old as he was y years ago; what is his present age?

3. (1) If $\frac{a}{b}, \frac{b}{c}, \frac{c}{d}, \frac{d}{e}$ are in proportion, prove that

$$\frac{b^4 + a^2c^2}{b^4 - a^2c^2} = \frac{d^4 + c^2e^2}{d^4 - c^2e^2}.$$

(2) If x, y, z are in arithmetical progression, prove that $x^2(y+z), y^2(z+x), z^2(x+y)$ are also in arithmetical progression.

4. Solve the equations:

$$(1) (x+y)(x^2-y^2) = a^3, (x-y)(x^2-y^2) = b^3.$$

$$(2) \sqrt{x} + \sqrt{2x-2} = \sqrt{5x+4}.$$

5. (1) If α, β are the roots of the equation $(x+a)(x+b) = c^2$,

$$\text{prove that } \frac{a+\alpha}{a+\beta} = \frac{b+\beta}{b+\alpha}.$$

(2) Prove that the roots of the equation

$$(a^2+ab)x^2+c^2x+ab-b^2=0$$

are rational if $a^2+b^2=c^2$.

6. The perimeters of two squares differ by 32 feet. If the area of the larger is 68 square feet less than the square on the diagonal of the smaller, find the area of the smaller.

7. In how many ways can six people be arranged in an order of merit? In how many of these orders will a certain person A be one place above a certain person B ?

8. A particle starts from a point O and moves along a straight line OA . If it is at the point P after t seconds, where $OP = s$ feet, and if s and t are always connected by the relation $s = 5t^2 + 3t$, find the distance it travels between the times 2 seconds and $2 + h$ seconds; find its average speed in this interval, and deduce its velocity at the end of two seconds.

152.

1. Prove that the expressions

$$\frac{1}{(1-ab)(1-a^2b^2)} \left\{ 1 - \frac{1}{1-a^2b} - \frac{1}{1-ab^2} \right\}$$

and

$$\frac{1}{(1-a^2b)(1-ab^2)} \left\{ 1 - \frac{1}{1-ab} - \frac{1}{1-a^2b^2} \right\} \text{ are equal.}$$

2. Solve the equations:

$$(1) \quad x^2y = (a-x)^3, \quad xy^2 = (b-y)^3.$$

$$(2) \quad \frac{2}{y+1} - \frac{6}{x+2} = 2x - 5y - 4 = 0.$$

3. If 16 is added to the product of four consecutive odd numbers, prove that the result is a perfect square.

4. (1) If the sums of the first n , $2n$ and $3n$ terms of an arithmetical progression are x , y , z , prove that

$$x = y - \frac{z}{3}.$$

(2) If a , b , c are in harmonical progression, prove that

$$\frac{a}{b+c}, \quad \frac{b}{c+a}, \quad \frac{c}{a+b} \text{ are in harmonical progression.}$$

5. A , who rides 4 miles an hour faster than B and 6 miles an hour faster than C , takes 50 minutes less than B and 90

154 TEST PAPERS IN ELEMENTARY ALGEBRA

minutes less than C to go from X to Y . Find the distance between X and Y .

6. (1) Prove that $\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = 0$ has equal roots if $a^2 + b^2 + c^2 = bc + ca + ab$.

(2) If α, β, γ are the roots of $x^3 - 3x^2 + 4 = 0$, prove that $\alpha^2 + \beta^2 + \gamma^2 = 9$; $\alpha^3 + \beta^3 + \gamma^3 = 15$; $\alpha^4 + \beta^4 + \gamma^4 = 33$.

7. In how many ways can two cards be selected from a pack of 52 cards? What is the chance of the ace of clubs and the ace of spades being next to each other in any pack?

8. If $(x^2 - 4x - 2) \cdot f(x) - (x^2 - 2x + 3) \cdot \phi(x) \equiv 1$, where $f(x)$ and $\phi(x)$ are functions of x of the first degree, find what $f(x)$ and $\phi(x)$ are.

153.

1. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$, prove that $\frac{a}{c}$ is the cube root of $\left(\frac{a}{d}\right)^2$.

2. If $f(x) \equiv \frac{(x^4 - 4x^3 + 8x - 4)^2}{(x^2 - 2x + 2)^4}$ prove that $f(x) = f\left(\frac{2}{2-x}\right)$.

3. Solve the equations :

(1) $\frac{x-2}{x-1} - \frac{2x-1}{x-2} + 1 = 0$.

(2) $16(x^2 - xy) = 18(xy - y^2) = 1$.

4. In how many ways can eight different books be arranged on a shelf if two particular books must be next to each other?

5. If α, β are the roots of the equation $3x^2 - 7x = 11$, form the equation whose roots are $\alpha + 2\beta - 1$ and $2\alpha + \beta - 1$.

6. If the quantities $\frac{aq}{q+ap}, \frac{bq}{q+bp}, \frac{cq}{q+cp}$ are in harmonical

progression for any one pair of values of p, q , prove that they are in harmonical progression for all values of p, q .

7. Three gamblers play cards; on the first game A loses to B and C as much as each of them has to start with; on the second game B loses to C , A what each then has; on the third game C loses to A and B what each then has; they now have 3£ each. How much had each to start with?

8. The line AB , x inches long, is divided internally at C and externally at D in the ratio $a:b$. Find the length of CD .

154.

1. Prove that the equations

$$x^2 - 2ax + b = 0 \quad \text{and} \quad x^2 - 2bx + a = 0$$

have a common root if $4a + 4b + 1 = 0$.

2. Express as powers of 2 the following: (1) 4^n ; (2) $\frac{1}{4}$; (3) $\sqrt[3]{32}$; (4) $8 \times 16^{n-2}$; (5) $4^{n+1} \times 8^{1-n} \div 16^{3-n}$.

3. (1) For what value of k has the equation

$$x^2 - (k+3)x + k + 6 = 0$$

equal roots?

(2) What is the condition that one root of the equation

$$x^2 - 2(a-b)x + a^2 + b^2 = 0$$

should be twice the other?

4. How many numbers greater than 4000 can be formed from the digits 7, 6, 3, 2, 0, no digit being repeated in the same number?

5. If the sum of the first $3n$ terms of an arithmetical progression, whose first term is a , is equal to the sum of the next $2n$ terms, prove that the common difference is $\frac{2a}{7n+1}$.

156 TEST PAPERS IN ELEMENTARY ALGEBRA

6. If $3x - y - 2z = 0$ and $2x + y - 4z = 0$, find the value of $\frac{x^2 + z^2}{xy}$.

7. A, B travel along the same road in the same direction at a miles an hour and b miles an hour respectively. When A is at a place O , B is m miles ahead. If B is overtaken h hours later at a place n miles from O , find h and n in terms of a, b, m .

8. Express $3(17a - 5b)(a - 3b)$ as the difference of the squares of two linear functions of a, b . Give two solutions.

155.

1. If α is so small that α^3 is negligible, prove that $1 + \alpha^2$ is one root of the equation $x^3 + \alpha x^2 - (\alpha + 3\alpha^2)x = 1$.

2. Simplify $\frac{\sqrt{x^2 + 1} - x}{\sqrt{x^2 + 1} + x} + \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1} - x}$.

3. In how many ways can eight men be arranged at a round table if two particular men (1) refuse to sit next to each other; (2) insist on sitting together?

4. If s_n denotes the sum of the first n terms of an arithmetical progression, prove that the common difference is equal to $s_n - 2s_{n+1} + s_{n+2}$.

5. If the sum of the squares of the roots of $2x^2 - 3x - 7 = 0$ is equal to the sum of the cubes of the roots of $x^2 - x - c = 0$, find the value of c .

6. If $(3x + 1)(y - 1)$, $(x + 2)(y + 3)$, $2x(y + 1)$ represent ascending consecutive positive integers, what integers are they?

7. If 7 is a factor of $2x - 11y$, prove that 49 is a factor of $10x^2 + 2xy + 5y^2$, where x and y are integers.

8. The resistance of the air to a projectile varies as the square of the diameter and the cube of the velocity. If the resistance to a projectile one inch in diameter at a velocity of 1000 feet per second is $2\frac{1}{2}$ lbs. weight, find the resistance to a projectile 16 inches in diameter travelling 1500 feet per second.

156.

1. If $\frac{1}{a} + \frac{1}{c} = \frac{1}{b}$, prove that $a^3 - b^3 + c^3 + 3abc = (a - b + c)^3$.

2. Prove that $\frac{3}{2}(\sqrt{3} + 1)^2 - 2(\sqrt{2} + 1)^2 = \sqrt{59 - 24\sqrt{6}}$.

3. Solve the equations:

$$(1) (x - 2y)(x - y) = 10, (x + 2y)(x + 5y) = 322.$$

$$(2) \frac{(x + a)(x + b)}{x + a + b} = \frac{(x + c)(x + d)}{x + c + d}.$$

4. There are four stations on a branch railway line. How many different pairs of single third-class tickets must be supplied for use on the line?

5. The n th term of a series is $3 \cdot 2^n + 5n - 2$; find the sum of the first n terms.

6. If α, β are the roots of the equation $x^2 - 2x - 5 = 0$, find the equation whose roots are $\alpha^2 - \alpha\beta$ and $\beta^2 - \alpha\beta$.

7. Three places A, B, C on a railway are passed by a train travelling k miles an hour at x minutes, y minutes, z minutes past twelve. A second train passes A and B at p minutes, q minutes past twelve; at what time will it pass C and at what rate is it travelling? Each train is supposed to travel at a uniform rate.

8. Eliminate t from the equations

$$x(1 + t) + y(1 - t) = a, \quad \frac{x - yt}{y + xt} = \frac{1 + t}{1 - t}.$$

157.

1. If $a^3 + b^3 + c^3 = 3abc$, prove that any one of the three quantities $a^2 - bc$, $b^2 - ca$, $c^2 - ab$ is a mean proportional between the other two.

2. Solve the equations :

$$(1) \frac{2}{x+3} = \frac{1}{x+2} + \frac{1}{x+6}.$$

$$(2) ax^2 - c^2x = a^2x - ac^2.$$

$$(3) \frac{y}{3} + \frac{z}{4} = \frac{z}{4} + \frac{x}{2} = \frac{x}{2} + \frac{y}{3} = 1.$$

3. If $\frac{1}{p}$ is the q th term of an arithmetical progression and if $\frac{1}{q}$ is the p th term, prove that the sum of pq terms is $\frac{1}{2}(1 + pq)$.

4. If $x^2 + 2hxy + y^2$ is equal to $\frac{1}{2}[(x + \alpha y)^2 + (x + \beta y)^2]$ for all values of x and y , form the equation whose roots are α and β , and find for what values of h these roots are equal.

5. Find the number of ways in which 5 boys and 3 girls may be arranged in a row so that (1) the girls may be all together ; (2) no two girls together.

6. If $f(x)$ is a rational integral function of x , prove that the remainder when $f(x)$ is divided by $(x - a)(x - b)$ is

$$\frac{1}{a-b}[\{f(a) - f(b)\}x + af(b) - bf(a)].$$

7. $ABCD$ are four points on a line such that AB , AC , AD are in harmonical progression ; prove that $\frac{AB}{BC} = \frac{AD}{CD}$.

8. Four people wish to be conveyed as quickly as possible to a place 5 miles distant. They can walk 4 miles an hour and have a motor that can go 18 miles an hour, but only holds two besides the driver. Two set off on foot ; the car takes the

other two a certain distance and sets them down to continue on foot and returns for the others. At what point must the car set down the first two in order that all may arrive together?

R. D. N. M'S HIGH SCHOOL,

158.

Bangalore.

1. If $x - 2y + 3z = 0$, express $8xy + 12yz - 12zx$ in the form $ax^2 + by^2 + cz^2$.

2. Find the H.C.F. of $x^3 + x^2 - 2$ and $x^4 + x^3 - x - 1$; find also their L.C.M.

3. Prove that $(x - 1)^2$ is a factor of $x^{100} - 100x + 99$.

4. The interior angles of a rectilinear figure are in arithmetical progression; the least is 120° and the common difference is 5° . Find the number of sides.

5. If a, b are positive, and if a is less than b , which is the greater, $\frac{a}{b}$ or $\frac{a+1}{b+1}$?

6. At a dinner party there are 5 ladies and 5 gentlemen; the host and hostess each occupy one end of the table. In how many ways can the table be arranged if no two ladies sit together?

7. A takes 1 hour longer to walk x miles than B takes to walk y miles, and A walks one mile in k minutes less than B does. How many miles an hour does B walk?

8. If a, b, c are consecutive integers, of which a is the least, prove that the value of $a^3 + 6b^2 - c^3$ is independent of the value of a .

159.

1. The formula $W = 3.875(D^2 - d^2)l$ gives the weight [W lbs.] of a piece of lead pipe, D and d being the outer and inner

160 TEST PAPERS IN ELEMENTARY ALGEBRA

diameters of the pipe in inches and l its length in feet. Find the weight when $D = 1.632$ ins., $d = .468$ in., $l = 12.4$ feet.

2. If x, y, z are in geometrical progression, prove that

$$x^2y^2z^2(x^{-3} + y^{-3} + z^{-3}) = x^3 + y^3 + z^3.$$

3. Find a symmetrical homogeneous function of the second degree in x, y, z which equals 5 when $x=0, y=1, z=1$ and equals 6 when $x=0, y=0, z=1$.

4. In how many ways can n men stand in a row if two particular men are excluded from being at either end, and if two other men must be placed together?

5. If the two values of x obtained from solving the equations $y = mx + c, y^2 = 4ax$ are equal, prove that $mc = a$.

6. An arithmetical and harmonical progression have each the same first term a and the same last term l and the same number of terms n . Prove that the product of the $(r+1)$ th term of the one and the $(n-r)$ th term of the other is independent of r .

7. If $\lambda = (a+x)(b-x) = (b+x)(c-x)$, find λ in terms of a, b, c .

8. Prove that (1) $(a^2 + b^2)(c^2 + d^2)$ and (2) $(a^2 + b^2)^3$ can be expressed as the sum of two squares.

160.

1. Prove that every prime number, except 2 and 3, can be expressed in the form $6p+1$ or $6p-1$, where p is an integer.

2. Solve the equations:

$$(1) \frac{\frac{1}{2}}{x + \frac{1}{2}} - \frac{\frac{1}{3}}{x + \frac{1}{3}} - \frac{\frac{1}{4}}{x + \frac{1}{4}} + \frac{\frac{1}{5}}{x + \frac{1}{5}} = 0.$$

$$(2) \sqrt{2x+1} - \sqrt{x+1} = \sqrt{3x}.$$

3. How many lines must be drawn to join every pair of n points, no three of which lie on a straight line?

4. The second, fourth and seventh terms of an arithmetical progression are in geometrical progression. Find the common ratio of the geometrical progression.

5. (1) Prove that the roots of the equation $ax^2 + bx + c = 0$ are rational if $b - ac = 1$.

(2) Prove that the roots of the equation

$$x(a + b - \frac{1}{2}x) = a^2 + b^2$$

are imaginary.

6. If a cricketer scores a runs, his average will rise by c ; if he scores b runs, it will fall by c . What is his present average?

7. A train travelling 42 miles an hour passes a cyclist, going in the same direction, in 9 seconds. If the cyclist had been going in the opposite direction, the train would have passed him in 5 seconds. Find the length of the train in yards.

8. If $\frac{x+y}{x-y} = \frac{3}{4}$, find the value of $\frac{x^2 - xy + y^2}{x^2 + xy + y^2}$.

PAPERS 161-170 include

Partial fractions; easy probability.

161.

1. If $x^3 = 2$, prove that $(l + mx + nx^2)(3 - x + x^2)$
 $= 3l + 2m - 2n - (l - 3m - 2n)x + (l - m + 3n)x^2$.

2. (1) If

$x^2 - 3x - 7 \equiv a(x-1)(x-2) + b(x-2)(x-3) + c(x-3)(x-1)$.
 for all values of x , find the values of a , b , c .

(2) Given that $(b-c)^5 + (c-a)^5 + (a-b)^5$
 $\equiv (b-c)(c-a)(a-b)[l(a^2 + b^2 + c^2) + m(bc + ca + ab)]$,
 find the values of l and m .

162 TEST PAPERS IN ELEMENTARY ALGEBRA

3. If $x^2 = (a+b)y + ab$ and $y^2 = (a+b)x + ab$ and $x \neq y$, prove that $a^2 = (x+y)b + xy$.

4. If the second term of an arithmetical progression is three times the first term, prove that the sum of n terms is proportional to n^2 .

5. Find the surface and the volume of a cuboid, the lengths of whose edges in inches are the roots of the equation

$$x^3 - 9x^2 + 12x - 4 = 0.$$

6. Put into partial fractions :

$$(1) \frac{x-1}{(x+1)^2}; \quad (2) \frac{x-2}{(x+1)^2(x-1)}.$$

7. Find two numbers such that their product, their difference and the excess of the square of the larger over twice the square of the smaller are all equal.

8. If $m^2x - my + a = 0$ and $n^2x - ny + a = 0$,
and $m - n = c(1 + mn)$,
prove that $y^2 - 4ax = c^2(x + a)^2$.

162.

1. If $\frac{1}{a+b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{b+c}$, prove that

$$\frac{1}{(a+b)^{2n+1}} + \frac{1}{c^{2n+1}} = \frac{1}{a^{2n+1}} + \frac{1}{(b+c)^{2n+1}}.$$

2. (1) If $(x+y+z)^3 \equiv \Sigma x^3 + p\Sigma x^2y + qxyz$, find the values of p, q .

(2) If $8x^6 - 36x^5 + 66x^4 - ax^3 + bx^2 - 9x + c$ is a perfect cube, find the values of a, b, c .

3. Put into partial fractions $\frac{2x^2 - 5}{(x-1)(x-2)^2(x+3)}.$

4. Solve the equations :

$$(1) (x^{\frac{1}{2}} + a^{\frac{1}{2}})^{\frac{1}{2}} = (x^{\frac{1}{3}} + a^{\frac{1}{3}})^{\frac{1}{2}};$$

$$(2) yz + ax = zx + ay = xy + az = b^2.$$

5. A tank can be filled by two pipes ; one takes 8 hours longer than the other, and both pipes together take 11 hours 40 minutes. Find how long each pipe takes separately.

6. If the equations $lx + my = 1$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, when solved for x and y , have only one distinct solution, prove that $a^2l^2 + b^2m^2 = 1$.

7. (1) In how many ways can four people be selected from nine? What is the chance of one particular person being chosen?

(2) How many numbers greater than 5000 can be formed with the digits 8, 6, 4, 3, 0?

8. Find the sum of $n + 1$ terms of a geometrical progression, if a , b are the first and last terms.

Sum to n terms the series $1 + 11 + 111 + 1111 + \dots$

163.

1. Find the square root of

$$(x-1)^4 + 2(x^4+1) - 2(x^2+1)(x-1)^2.$$

2. If the roots of $ax^2 + bx + c = 0$ are twice the roots of $bx^2 + cx + a = 0$, find the first equation.

3. In how many ways can three cards be selected from a pack of 52 cards? In how many ways will one of these cards be (1) any ace, (2) the ace of spades?

4. Express in partial fractions $\frac{7x-8}{(x+1)(4x-1)}$.

5. If the sum of n terms of a series is $3n^2 - 2n$ for all values of n , find the first, second, third and n th terms.

6. Solve the equations :

$$(1) \frac{2}{x-3} + \frac{3}{x-2} + 1 = 0. \quad \cdot \cdot$$

$$(2) xy = x + y + 7, \quad yz = y + z + 23, \quad zx = z + x + 11.$$

7. A man travels 154 miles by train from A to B , and he returns to A by a train which goes 9 miles an hour faster, thus taking one hour less. Find the rates of the two trains.

8. Five men sit down at a round table. What is the chance that two particular men sit together?

164.

1. Simplify

$$\left(x^2 + x + 1 + \frac{1}{x} + \frac{1}{x^2}\right)^2 - \left(x^2 - x + 1 - \frac{1}{x} + \frac{1}{x^2}\right)^2 - 4\left(x^3 + \frac{1}{x^3}\right). \quad \cdot$$

2. Solve the equations :

$$(1) 1 + \frac{2x-5}{5+x} = \frac{4}{x-1}. \quad \cdot$$

$$(2) 2\sqrt{x+a} + \sqrt{x-a} = 3\sqrt{x}. \quad \cdot$$

3. Express in partial fractions $\frac{3x+1}{(x-1)(x^2-1)}$.

4. (1) If $x-a$, $y-a$, $z-a$ are in geometrical progression, prove that $2(y-a)$ is the harmonic mean between $y-x$ and $y-z$.

(2) If, in an arithmetical progression, k times the k th term equals n times the n th term, prove that the $(k+n)$ th term is zero.

5. Find the condition that the ratio of the roots of the equation $x^2+ax+b=0$ is equal to that of the roots of $x^2+bx+a=0$.

6. From five men a tennis four is to be made up. Each man spins his racquet, the odd man being left out. What is the chance that the first spin will be decisive?

7. Three persons A, B, C have 3£ between them. A gives $\frac{1}{3}$ of what he has to C , B then gives $\frac{1}{4}$ of what he has to A , and they end by having equal amounts. How much had A at first?

8. A cistern with a leak in the bottom can be filled by one pipe in x minutes and by two pipes in y minutes. In how many minutes can it be filled by three pipes, all the pipes being the same size?

165.

1. Prove that $(x^2 - 2xy - y^2)^3 - (x^6 - 8x^3y^3 - y^6)$ is equal to $3xy(x - y)(x + y)(2y - x)(2x + y)$.

2. Put into partial fractions $\frac{5x^2 - 11x + 8}{(1 - x)(2 - x)^2}$.

3. If the roots of the equation

$$\left(1 - q + \frac{p^2}{2}\right)x^2 + p(1 + q)x + q(q - 1) + \frac{p^2}{2} = 0$$

are equal, prove that $p^2 = 4q$ or $p^2 + (q - 1)^2 = 0$.

4. Points $O, A_1, A_2, A_3, \dots, A_n, \dots$ are marked on a line in such a way that $OA_n = 4OA_{n-1} - 3OA_{n-2}$ for all values of n from 3 upwards. Prove that $A_1A_2, A_2A_3, A_3A_4, \dots$ are in geometrical progression. If $OA_1 = p, OA_2 = q$, find the length of $A_{n-1}A_n$ in terms of p, q .

5. (1) If $\frac{a}{b} = \frac{c}{d}$, prove that $\frac{a+c}{b+d} = \frac{a-c}{b-d}$.

(2) Solve the equation

$$\frac{x^2 + 3x - 1}{3x^2 + 9x + 2} = \frac{x^2 - 3x - 3}{3x^2 - 9x - 4}.$$

166 TEST PAPERS IN ELEMENTARY ALGEBRA

6. If α, β are the roots of the equation $px^2 + qx + r = 0$, find the relation between p, q, r if $\frac{2}{\alpha} + \frac{1}{\beta} = 1$.

7. A proper fraction, the difference between whose numerator and denominator is 2, is such that if unity is added to the numerator, the result is the same as if 9 is added to both numerator and denominator. Find the fraction.

8. How many integers are there less than 1000 which have neither 2 nor 7 as a factor?

166.

1. The time, T seconds, of a complete vibration of a flat spiral spring when loaded with a mass of M grammes is given by

$T = 2\pi \sqrt{\frac{M + \frac{1}{3}m}{\pi b^4 n \div 2la^2}}$, where b is the radius of the cross-section of the wire, a the radius of the cylinder on which the spring is wound, l the length of spring, all expressed in cms.; the mass of the spring is m grammes and n is a constant depending on the material. Find T to 3 significant figures if $l = 300$, $a = 1.53$, $b = .18$, $M = 1000$, $m = 315$, $n = 8.1 \times 10^6$.

2. If g is the geometric mean between a and b , prove that the ratio of the arithmetic and harmonic means of a, g is equal to the ratio of the arithmetic and harmonic means of g, b .

3. Find the partial fractions of $\frac{x^4 + x^2}{x^2 - 1}$.

4. Sum to 20 terms:

$$(1) (a + 2b) + (2a + b) + (3a) + (4a - b) + \dots$$

$$(2) (1 + 2) + (2 + 4) + (4 + 6) + (8 + 8) + (16 + 10) + \dots$$

5. Solve the equations :

$$(1) \frac{(x-2)^2}{2} + \frac{(2x-1)(x+5)}{3} = \frac{7x-1}{2}.$$

$$(2) 3x^2 + 5xy + y^2 + 3 = 0, \quad 2x + 3y + 1 = 0.$$

6. Two places A , B are 4 miles apart on a river. A man, who can row five miles an hour in still water, finds that it takes 36 minutes longer to row from A to B than it does from B to A . Find the speed of the stream.

7. A and B draw cards from a pack. A draws first and gets the six of hearts. B then draws a card; what is his chance of getting a higher card? [Aces count as lower.]

8. What is the condition that $x^3 + ax + b$ is a factor of $x^4 + x^3 + ax^2 + cx + b$?

167.

1. If $x^2 + y^2 = z^2$, prove that $\frac{y-z+x}{z-x+y} = \frac{x-y+z}{x+y+z}$.

2. Solve the equations :

$$(1) \frac{a^2(x-b)}{a-b} + \frac{b^2(x-a)}{b-a} = x^2.$$

$$(2) x^2 + y^2 = x + y = 2.$$

3. Find the sum of all integers between 100 and 800 which are not divisible by 7. [100 and 800 are excluded.]

4. Simplify (1) $\frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{2+\sqrt{3}} - \frac{1}{\sqrt{2}-1}$.

$$(2) \frac{x^{3a} - y^{3b}}{x^{2a} - y^{2b}} - (x^a + y^b).$$

168 TEST PAPERS IN ELEMENTARY ALGEBRA

5. Find a, b, c if $(1+x)(a+bx) - (1-x)(b+cx)$ is equal to 1 for all values of x .

6. If a, m, b are in arithmetical progression and a, n, b in geometrical progression, prove that $\frac{m}{\sqrt{m^2 - n^2}} = \frac{a+b}{a-b}$.

7. If increasing the price of oranges one penny per dozen results in two less being bought for one shilling, what is the price per dozen?

8. There are n points in a plane, no three of which are collinear. How many triangles can be formed having these points as vertices?

168.

1. The following values were obtained by observation :

$x = 0$	1	2	3	4	5	...
$y = \cdot 7$	1·4	2·7	4·8	7	10	...

It is probable that one of these is wrong, which is it likely to be? and prove that the others satisfy an equation of the form $y = ax^2 + bx + c$, and find the values of a, b, c to one place of decimals.

2. Simplify $a + bx + cx^2 - \frac{ab + (b^2 - ac)x}{b - cx}$.

3. If $\frac{a}{b} = \frac{b}{c}$, prove that $(a+b)(c-b) = (b+c)(b-a)$.

4. Find the number of ways in which nine different things can be distributed into sets of three.

5. How many terms of an arithmetical progression, whose first term is -7 and common difference $\frac{3}{5}$, must be taken so as to add up to -26 ?

6. Solve the equations :

$$(1) \ x = 5(x - y + 7), \quad y = 7(y - x - 5).$$

$$(2) \ \sqrt{2x+3} - \sqrt{3x-5} = \sqrt{4x-3} - \sqrt{x+1}.$$

7. A reduction in speed of 3 miles an hour makes a difference of 50 hours in duration of a voyage of 3000 miles. Find the speed of the steamer.

8. A man will be x years old in the year x^2 . When was he born?

169.

1. Prove that $\frac{1}{(y-z)^2} + \frac{1}{(z-x)^2} + \frac{1}{(x-y)^2}$ is a perfect square.

2. If $f(x)$ is a function of x of the first degree, express $\frac{f(x)}{(x-a)^2(x-b)}$ in partial fractions.

3. A man has 12 relations, 7 ladies and 5 gentlemen ; his wife has 12 relations, 5 ladies and 7 gentlemen. In how many ways can they arrange a dinner party for six of the man's relatives and six of the wife's, the number of ladies and gentlemen present being equal?

4. Each digit of a number of $2n$ digits is one. Prove that the integral part of its square root is a number of n digits, each of which is 3, and that the remainder is a number of n digits, each of which is 2.

5. If p, q, r are consecutive terms in a series whose n th term is $an^2 + b$, prove that $p - 2q + r = 2a$.

6. The sum of the roots of a quadratic is 5, and the sum of the squares of the roots is 21. What is the equation?

7. The area of one rectangular field is 560 sq. yards less than that of another. The second is 5 yards longer and

170 TEST PAPERS IN ELEMENTARY ALGEBRA

6 yards wider than the first, and the width of the second is 2 yards more than the length of the first. Find the area of each.

8. Prove that ${}^nC_r + {}^nC_{r-1} = \frac{n+1}{r} \cdot {}^nC_{r-1}$.

170.

1. Prove that $(\sqrt{5} + 1)^{\frac{1}{2}} \cdot (\sqrt{5} + 2)^{\frac{1}{2}} = \frac{1}{2}(3\sqrt{2} + \sqrt{10})$.

2. Express in partial fractions $\frac{2x^2 + 3x + 1}{(x-2)^3(x-3)}$.

3. If the first term of an arithmetical progression is 13, and if the sum of 15 terms is 510, what is the 15th term?

4. Prove that $a+b$ is a factor of $(a+b+c)^3 - (a^3 + b^3 + c^3)$, and then factorise this expression.

5. Solve the equations :

(1) $2xy - x^2 = 24, \quad 2y^2 - xy = 30.$

(2) $(2a - 3b)(x - 2a) = (b - 3a)(x + 3a - 4b).$

6. A , who can ride a miles an hour and walk b miles an hour, and B , who can ride c miles an hour and walk d miles an hour, are at leave out n miles from school with only one bicycle available. In order that they may stay as late as possible, a servant, who can ride e miles an hour, has been sent on ahead on foot. Show that if they arrange it properly

they can both get back in $\frac{n[ac - bd + e\{a + c - b - d\}]}{ac(b + d + e) - bd(a + c + e)}$ hours.

Prove that their time must always be greater than $\frac{n(a + c - b - d)}{ac - bd}$ hours.

7. It is 2 : 1 against a certain man leaving his umbrella at

any house at which he calls. What is the chance that he has his umbrella with him after paying 3 visits?

8. A man buys a horse for x £ and sells it for £20. 11s. 3d., making thereby a profit of x per cent. Find x .

ADAM'S HIGH SCHOOL

PAPERS 171-180 include

Bangalore.

Practice in manipulation; hard miscellaneous equations; logarithms to any base; maxima and minima.

171.

1. If $x^2 - yz = a$, $y^2 - zx = b$, $z^2 - xy = c$, $xy + yz + zx = 0$, prove that $x^2y^2z^2 = a^2b^2c^2(a + b + c)^{-3}$.

2. (1) Prove that $2(1 + 6x - 2x^2) = 11 - (3 - 2x)^2$, and deduce the least value of $2x^2 - 6x - 1$, if x is real.

(2) Find the greatest value of $3 + x - x^2$, if x is real.

3. If $\frac{x(y+z-x)}{\log x} = \frac{y(z+x-y)}{\log y} = \frac{z(x+y-z)}{\log z}$, prove that $y^z \cdot z^y = z^x \cdot x^z = x^y \cdot y^x$.

4. Solve the equations:

$$(1) (x-2)^{\frac{1}{3}} + (x-3)^{\frac{1}{3}} + (5-2x)^{\frac{1}{3}} = 0.$$

$$(2) \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = \frac{x}{a^2} + \frac{y}{b^2} + \frac{z}{c^2} = 0; \quad x + y + z = (b-c)(c-a)(a-b).$$

$$(3) 2x + 5 = 2(x^2 + 5x + 6)^{\frac{1}{4}}.$$

5. If $\frac{x}{a} = y + z$, $\frac{y}{b} = z + x$, $\frac{z}{c} = x + y$, prove that

$$(1) ab + bc + ca + 2abc = 1; \quad (2) \frac{x^2}{a(1-bc)} = \frac{y^2}{b(1-ca)} = \frac{z^2}{c(1-ab)}.$$

6. A starts from X and walks to Y , and B , leaving Y at the same time as A leaves X , walks to X ; A arrives at Y

172 TEST PAPERS IN ELEMENTARY ALGEBRA

20 minutes after passing B and B arrives at X 45 minutes after passing A . How long does each take over the journey?

7. In how many ways can three ladies and three gentlemen be arranged at a dinner table if ladies and gentlemen are placed alternately?

8. If the equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ has equal roots, prove that a, b, c are in arithmetical progression.

172.

1. If $\frac{b-c}{y-z} + \frac{c-a}{z-x} + \frac{a-b}{x-y} = 0$, prove that

$$(b-c)(y-z)^2 + (c-a)(z-x)^2 + (a-b)(x-y)^2 = 0.$$

2. Factorise

$$(1) (b+c)(c+a)(a+b) + abc.$$

$$(2) (ab+cd+b^2-d^2)^2 - (ad+bc)^2.$$

$$(3) (a+b+c)^4 - (b+c)^4 - (c+a)^4 - (a+b)^4 + a^4 + b^4 + c^4.$$

3. Solve the equations:

$$(1) x^2 + \frac{1}{y} = y^2 + \frac{1}{x} = a, \text{ assuming that } x \neq y.$$

$$(2) (x+4)(x+7)(x+8)(x+11) + 20 = 0.$$

$$(3) x^x - 4x^{-x} = 3, x \text{ being real.}$$

4. If $ax^3 + bx^2 + cx + d$ is divisible by $x^2 + e^2$, prove that a, b, c, d are proportionals.

5. (1) Find an equation whose roots are the squares of the roots of the equation $x^2 - 5x + 2 = 0$.

(2) Find the condition that the roots of the equation $x^3 + px^2 + qx + v = 0$ are in geometrical progression.

6. (1) If the sum of n terms of an arithmetical progression is always equal to $2n - 7n^2$, find the n th term.

- (2) A, G, H are the arithmetic, geometric and harmonic means between x and y ; H' is the harmonic mean between A, G , and A' is the arithmetic mean between G, H . Prove that $G^2 = A' \cdot H'$.

7. (1) Prove that $\log_a b \cdot \log_b c \cdot \log_c a = 1$.

(2) Solve the equation :

$$\log(x^2 - 3x + 2) - \log(x^2 + 5x + 6) = \log(x^2 - 1) - \log(x^2 + x - 2).$$

8. A straight road leads through a forest to a point Y ; a man is at a point X in the forest some way from the road; he can walk 2 miles an hour in the forest and 5 miles an hour along the road. If he strikes the road at Z , where $XZ = YZ$, he takes $3\frac{1}{2}$ hours to get from X to Y . If he walks straight to the road, his times in the forest and along the road are equal. How far is he from the road?

173.

1. If $\frac{a}{b} = \frac{c}{d}$ and if a, b, c are in arithmetical progression, prove that b, c, d are in harmonical progression.

2. If $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$, prove that

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

3. Solve the equations :

$$(1) \left(\frac{x+a}{x-a} \right)^{\frac{1}{3}} + \left(\frac{x-a}{x+a} \right)^{\frac{1}{3}} = 3.$$

$$(2) x^{2(x+y)} = y, \quad y^{x+y} = x^2 \text{ to one place of decimals (positive solutions only).}$$

4. Simplify $\log \frac{351}{639} + 2 \log \frac{91}{110} - 3 \log \frac{39}{110}$.

5. Express $4x^2 + 12x + 15$ in the form $(ax + b)^2 + c$, and hence determine the least value of this expression, if x is real.

174 TEST PAPERS IN ELEMENTARY ALGEBRA

6. A structure may be built at a cost of $\pounds c_1$ to last for n_1 years, or at a cost of $\pounds c_2$ to last n_2 years, and it is renewed indefinitely whenever required. Making no allowance for repairs, prove that the ultimate cost of each design is the same if $\frac{c_1}{c_2} = \frac{1 - R^{-n_1}}{1 - R^{-n_2}}$, where R is the amount of 1£ in 1 year, compound interest being reckoned yearly.

7. If a is a root of $x^4 + x^3 - 4x^2 - 4x + 1 = 0$, prove that $2 - a^2$ is another root.

8. Four letters are written and four envelopes addressed. In how many ways can all the letters be placed in the wrong envelopes?

174.

1. (1) Divide $x^{\frac{2}{3}} + 4y$ by $x^{\frac{1}{6}}y^{-\frac{1}{4}} + 2 + 2x^{-\frac{1}{6}}y^{\frac{1}{4}}$.

(2) Find the square root of

$$8(1+a)(1+b)(1+ab) + (1-a)^2(1-b)^2.$$

2. If $2 > a > 0$, prove that the equation

$$(x+3)(1-ax) - (1+a)(x+3) = 2$$

has no real roots.

3. If the first and second terms of an arithmetical progression are $\log x$, $\log y$, prove that the n th term is $\log \left(\frac{y^{n-1}}{x^{n-2}} \right)$.

4. Solve the equations:

$$(1) (x+3)(x+5)(x+9)(x+11) = 45.$$

$$(2) \frac{x^3}{y} - \frac{y^3}{x} = \frac{15}{2}, \quad \frac{x}{y} - \frac{y}{x} = \frac{3}{2}.$$

5. Prove that ${}^nC_{r+1} + {}^nC_{r-1} = 2 \cdot {}^nC_r + {}^{n+2}C_{r+1}$.

6. The chance that A defeats B at a certain game is p and that A defeats C is q . Find the chance that B defeats C .

7. If one pair of opposite sides of a square lawn are each diminished by 25 feet and the other sides increased by 20 per cent., the new area of the lawn is less than the old by 1000 sq. feet. Find the length of a side of the lawn.

8. Write down the n th terms of the following series :

(1) $3, 1, \frac{3}{5}, \frac{3}{7}, \frac{1}{3}, \dots$ (2) $1 \times 3, 2 \times 4 \times 6, 3 \times 5 \times 7 \times 9, \dots$

(3) $1 \times 1, 3 \times 4, 5 \times 9, 7 \times 16, 9 \times 25, \dots$

175.

1. If $x^2 = y^2 + z^2 - 2lyz$, $y^2 = z^2 + x^2 - 2mzx$, $z^2 = x^2 + y^2 - 2nxy$, prove that $\frac{1-l^2}{x^2} = \frac{1-m^2}{y^2} = \frac{1-n^2}{z^2}$.

2. Solve the equations :

(1) $yz^2 = 75, zx^2 = 20, xy^2 = 18.$

(2) $x(1-y^2) = 2y, y(1-x^2) = 2x.$

3. If $\phi(x) = a + \frac{\log(\log x)}{\log r}$, prove that $\phi(x^r) = 1 + \phi(x)$.

4. Find the sum of the first n brackets of the series :

$$(1) + (3 + 5 + 7) + (9 + 11 + 13 + 15 + 17) \\ + (19 + 21 + 23 + 25 + 27 + 29 + 31) + \dots$$

5. If x, y, z are real, prove that

(1) $x^2 + y^2 > 2xy.$

(2) $x^2 + y^2 + z^2 > xy + yz + zx.$

6. Three trains of lengths a, b, c feet are travelling with uniform velocities u, v, w feet per second in the same direction on equidistant parallel rails with their rear-most carriages in a straight line. Prove that the trains may all be cut by a straight line for a time $\frac{a+c}{2v-u-w}$ seconds or for a time $\frac{2b}{u+w-2v}$ seconds according as $2v$ is greater or less than $u+w$.

7. Ten people meet to play tennis. In how many ways can two sets of four players each be formed?

8. If $x^3 = x^2 + 1$, prove that $x^5 = 2x^2 + x + 1$.

176.

1. Prove that $(a + b - 2c)^3 + (b + c - 2a)^3 + (c + a - 2b)^3$ is equal to $3(a + b - 2c)(b + c - 2a)(c + a - 2b)$.

2. Find the factors of

$$(1) 12x^2 - 7x - 12. \quad (2) x^4 + 4.$$

$$(3) 2x^3 - 5x^2 + 36.$$

3. Solve the equations:

$$(1) (x + y)\sqrt{x - y} = a^3, \quad (x - y)\sqrt{x + y} = b^3.$$

$$(2) \frac{x + a}{x - a} + \frac{x + b}{x - b} = \frac{2x}{x - a - b}.$$

4. Prove that the arithmetic mean between the harmonic and geometric means of a and b is $\frac{\sqrt{ab}(\sqrt{a} + \sqrt{b})^2}{2(a + b)}$.

5. If p, q, r are the logarithms of a, b, c , prove that $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$.

6. A mixture whose weight is 100 grams is composed of 25 % bismuth, 40 % tin, 35 % lead. How much tin and how much lead must be added to obtain a mixture containing 10 % bismuth, 50 % tin and 40 % lead?

7. ABC is a triangle right-angled at B , $AB = 3''$, $BC = 5''$. Find a point P on BC such that $\frac{1}{4}AP^2 + PC^2$ is a minimum.

8. Find a number of two digits which is equal to twice the product of its digits.

177.

1. If $f(x) \equiv 5 \cdot 2^x + 3x - 2$, prove that

$$f(x) - 4f(x-1) + 5f(x-2) - 2f(x-3) = 0.$$

2. If l_1, l_2, l_3 are the n th, $2n$ th, $3n$ th terms of a geometrical progression, prove that $l_2^2 = l_1 \cdot l_3$.

3. Solve the equations :

$$(1) (x+2y)(2x+y) = 20, \quad 4x(x+y) = 16 - y^2.$$

$$(2) x^x - 4x^{-x} = 3.$$

4. In how many ways can a tennis four, two ladies and two gentlemen, be chosen from five married couples, the gentlemen being put on opposite sides, assuming that in no case a husband and wife are in the same set.

5. For what real value of x is the expression $1 + \frac{2}{x} + \frac{3}{x^2}$ least?

6. If in the County Championship m points are given for a win and $-n$ for a loss, find the ratio of the number of points gained to the number of finished matches in the case of a county which wins k matches and loses l matches. Show that the order of the counties determined in this way is unaffected by the values of m and n .

7. Find the condition that the equation

$$\frac{l}{x-a} + \frac{m}{x-2a} + \frac{n}{x-3a} = 0$$

has equal roots.

8. If $\frac{1}{a} + \frac{1}{c} = \frac{1}{2b-a} + \frac{1}{2b-c}$, prove that $2b$ is either the arithmetic mean between $2a$ and $2c$ or the harmonic mean between a and c .

178.

1. If $x + y = a + b + c$ and

$$x(x - a)(x - b)(x - c) = y(y - a)(y - b)(y - c)$$
and $x \neq y$, prove that $x^3 + y^3 = a^3 + b^3 + c^3$.
2. If a, b, c are in arithmetical progression, b, c, d in geometrical progression and c, d, e in harmonical progression, prove that a, c, e are in geometrical progression.
3. If the equations $ax^2 + bx + c = 0$, $cx^2 + ax + b = 0$ have a common root, where a, b, c are real and unequal and $c \neq 0$, prove that $a + b + c = 0$.
4. If $u_n = 3 \cdot 2^{n-1} + 4n - 7$, find the value of

$$u_1 + u_2 + u_3 + \dots + u_n.$$
5. Prove that $(\log_b a) \cdot \log_a (\log_a b) + (\log_b \log_b a) = 0$.
6. Solve the equations :

$$(1) \quad x^2 - 15 = y^2 - 10 = xy.$$

$$(2) \quad ax + by = a^2 - b^2, \quad x^2 + y^2 = a^2 + b^2.$$
7. In a certain election there are two candidates A and B ; 20 more electors vote for B than A ; but 1 % of A 's votes and $1\frac{1}{4}$ % of B 's votes being disallowed, A is elected by a majority of 2. Find the number of voters.
8. A party of 6 men, on saying good-night, shake hands with each other. How many hand-shakings are there?

179.

1. Simplify

$$(x^2 + 3)(x^2 - 3x + 3)(x^2 + 3x + 3)$$

$$- (x^2 + \frac{1}{3})(x^2 + x + \frac{1}{3})(x^2 - x + \frac{1}{3}).$$
2. If A is the arithmetic mean and G the geometric mean between p and q , prove that $p^3 + q^3 = 8A^3 - 6AG^2$.

3. The sum of $2n$ terms of an arithmetical progression, whose first term is a , is 3 times the sum of n terms. Prove that the common difference is $\frac{2a}{n+1}$.

4. Solve the equations :

$$(1) \quad xy + xy^2 = 12, \quad x + xy^3 = 18.$$

$$(2) \quad xyz = 231, \quad xyw = 420, \quad yzw = 1540, \quad xzw = 660.$$

5. If a, b are consecutive integers, prove that

$$b^3 - a^3 = 3ab + 1.$$

6. The n th number of a set of numbers is $an^2 + bn + c$, where a, b, c are constants. If p, q, r, s are any four consecutive numbers in this set, prove that $p - s = 3(q - r)$.

7. A man starts to bicycle 15 miles, but after he has covered 12 miles his bicycle breaks down and he has to walk. He takes altogether 1 hour 52 minutes. Had his bicycle broken down after he had been travelling one hour, he would have taken two hours in all. How fast does he ride?

8. AB is a straight line of length one foot. Find a point P on it such that $3AP^2 + PB^2$ is a minimum.

180.

1. If $a + b + c = 0$, prove that

$$(b^2 - c^2)(b - c)^2 + (c^2 - a^2)(c - a)^2 + (a^2 - b^2)(a - b)^2 = 0.$$

2. Find the partial fractions of $\frac{x^2}{(x-1)^3(x^2+1)}$.

3. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, prove that

$$\frac{a^m}{b^{m-1}} + \frac{c^m}{d^{m-1}} + \frac{e^m}{f^{m-1}} = \frac{(a+c+e)^m}{(b+d+f)^{m-1}}.$$

180 TEST PAPERS IN ELEMENTARY ALGEBRA

4. If α, β are the roots of the equation $ax^2 + bx + c = 0$, prove that $\frac{\beta}{a\alpha + b} + \frac{\alpha}{a\beta + b} = -\frac{2}{a}$.

5. Find the values of a, b if $x^2 - x + 1$ is a factor of $x^5 - x^3 + ax + b$.

6. If $a = x^3, b = x\sqrt[3]{x}, c = x^{-\frac{3}{2}}$, express $\frac{b^2c}{a^3}$ as a power of x .

7. Solve the equation $(ax)^{\log a} = (by)^{\log b}, b^{\log x} = a^{\log y}$.

8. Two persons A and B walk from P to Q , and a third person C walks from Q to P . A and C start at one o'clock; B starts at 1.10 and overtakes A at 1.30; B meets C at 2.20 and A meets C at 2.30. Compare the rates at which A, B, C walk.

PAPERS 181-190 include

Algebraic form; symmetry; imaginaries; limitation of values of a function; harder graphs.

181.

1. If $\frac{1}{y-z} + \frac{1}{z-x} + \frac{1}{x-y} = 0$, prove that

$$(y-z)^3 = (z-x)^3 = (x-y)^3.$$

2. If $(x+1)(y-1) = (y+1)(z-1) = (z+1)(x-1) = \lambda - 1$ and $x + y + z = 3a$, prove that $\lambda = a^2$ or -3 .

3. Factorise (1) $\sum_{x, y, z} x(y-z)(y+z-x)^4$.

$$(2) (1+a)^2 - 2(1+a^2)b^2 + (1-a^2)b^4.$$

$$(3) (a+b-c)(bc+ca-ab) - abc.$$

4. If $f(x)$ denotes the algebraically greatest integer in x , draw the graphs of the functions $f(x)$ and $x - f(x)$.

5. Solve the equations :

$$(1) \quad x^2y - xy^2 = 6, \quad x^3 - y^3 = 26.$$

$$(2) \quad a(b - c)x^2 + b(c - a)x + c(a - b) = 0.$$

6. Find a value of a such that the expression $\frac{x^2 - 8x + a^2}{x^2 + 8x + a^2}$ is capable of all values between 5 and $\frac{1}{5}$, but no others.

7. (1) Prove that if n is not divisible by 100, the number of days in the year n A.D. is

$$365 + \frac{1}{4}\{1 + (-1)^n\} \{1 + (\sqrt{-1})^n\}. \quad [\text{Hardy.}]$$

(2) If $\frac{2 - 3\sqrt{-1}}{1 + 2\sqrt{-1}} = a + b\sqrt{-1}$, where a, b are real, find the values of a and b .

8. The amount of a work a certain labourer does per hour varies directly as the square root of his pay per hour and inversely as the square root of the number of hours per day he works. A piece of work takes him 4 days if he works 8 hours a day and is paid 9 pence per hour; how long will it take if he works 6 hours a day and is paid one shilling an hour? How is the cost to the employer affected by varying the arrangement made with the labourer?

182.

1. Find a symmetrical homogeneous function of the second degree in x, y, z which equals 1 when $x=0, y=1, z=-1$ and equals 2 when $x=0, y=0, z=1$.

2. Prove that the sum of the twentieth powers of the roots of the equation $x^4 + ax + b = 0$ is $50a^4b^2 - 4b^5$.

182 TEST PAPERS IN ELEMENTARY ALGEBRA

3. It is known that the sum of n terms of the series whose r th term is $(-1)^{r-1} \cdot r^4$ is $(-1)^{n-1} \cdot \frac{1}{2}n(n^3 + an^2 + b)$, where a, b are constants. Find the values of a, b .

4. Find three positive numbers in arithmetical progression such that three times the third exceeds the product of the other two by 2 and the square of the third exceeds three times the product of the other two by 16.

5. Simplify $\frac{1}{(1+\sqrt{-1})^4} + \frac{1}{(1-\sqrt{-1})^4}$.

6. If $\frac{a}{b} = \frac{c}{d}$, prove that $\frac{ac - c^2}{b - d} = \frac{d(a^2 + c^2)}{b^2 + d^2}$.

7. Trace roughly the graph of the function $\frac{(x-1)(x-3)}{x-2}$.

8. Assuming that the horse-power required to propel a steamer of a given design varies as the square of the length and the cube of the speed, prove that a 2 % increase in length and a 7 % increase in horse-power will yield approximately a 1 % increase in speed.

183.

1. Prove that the value of

$$(x^2 + 3x + 9)(x^4 - 2x + 3) - (x^3 + 3x^2 + 9x - 1)(x^3 - 1)$$

is independent of x .

2. If x is real, prove that $\frac{x^2 - 4x - 20}{x - 7}$ cannot lie between 8 and 12.

3. If $2x = -1 + \sqrt{-3}$, prove that $x^8 + x^4 + 1 = 0$.

4. If $x = \alpha$ is a root of the equation $x^3 - 3x + 1 = cx(x - 1)$, prove that $1 - \frac{1}{\alpha}$ is also a root and that $\frac{1}{1 - \alpha}$ is the third root.

5. If $\sum_{r=1}^{r=n} f(r) = 3^{n+1}(n-1) + 3$ for all values of n , find the value of $f(r)$ in terms of r .

6. If $f(x)$ represents the greatest integer in x , draw roughly the graph of $x + f(x)$.

7. Four people draw cards from an ordinary pack. What is the chance that the cards are all the same suit?

8. A train travels 80 miles in 2 hours without stopping. The first 20 miles is level, the next 40 is uphill and the last 20 downhill. Its speed is 20 % less on the uphill and 20 % greater on the downhill than on the level. Find the speed of the train on the level.

184.

1. Find the factors of

(1) $x(x-2y)^3 - y(y-2x)^3$.

(2) $6x^3 - 13x^2 - 10x + 24$.

2. (1) Find the equation whose roots are x, y, z .

(2) Solve the equations:

$$x + y + z = a, \quad xy + yz + zx = -b^2, \quad xyz = -ab^2.$$

3. If a, b, c, d are in continued proportion, prove that

$$\frac{a}{d} = \frac{a(b^2 - c^2)}{b(c^2 - d^2)}.$$

4. If x is real, prove that $x^2 - 3x + 4$ is never less than $1\frac{3}{4}$.

5. What is the condition that the solutions of the equations $3x + 4y = 7$, $x^2 + y^2 = c^2$ are imaginary?

6. Trace roughly the graph of $\frac{(x-1)(x-3)(x-4)}{(x-2)(x-5)}$.

7. Prove that the n th term of the series

$(1 + 2) + (3 + 4 + 5 + 6) + (7 + 8 + 9 + 10 + 11 + 12) + \dots$
is $n(2n^2 + 1)$.

8. Two cities are a miles apart; A leaves one b hours before B leaves the other, and they meet c hours after A has started. If B had started b hours before A , they would have met $2c$ hours after B had started. Find the rate at which each walks.

185.

1. If $a = x^2 + 2yz$, $b = y^2 + 2zx$, $c = z^2 + 2xy$, find the square root of $a^2 + b^2 + c^2 - bc - ca - ab$.

2. For what values of x is $6x^2 + 7x$ greater than 3?

3. Rationalise the denominator of the fraction

$$\frac{3 - \sqrt{7}}{\sqrt{2} + \sqrt{3} + \sqrt{6} + \sqrt{7}}.$$

4. If the roots of the equation $x^3 + 2x^2 + q(x + 1) = 0$ are in harmonical progression, prove that $q = -4\frac{1}{2}$.

5. Solve the equations:

$$(1) \quad \frac{1}{x} + \frac{1}{y} = (x - 4)(y - 4) = 2.$$

$$(2) \quad \left(\frac{x-2}{x+1}\right)^3 = \frac{x-5}{x+4}.$$

6. If the q th term of an arithmetical progression is x and the r th term is y , prove that the sum of n terms is

$$\frac{n}{2(r-q)}[x(2r-n-1) - y(2q-n-1)].$$

7. If x is any positive whole number, prove that the sum of all whole numbers which are less than x and have no factor in common with x is divisible by x .

8. Two boys A , B do this paper. A tries every question, and the chance that he succeeds in any given question is $\frac{1}{4}$. The chance that B , who is rather lazy, tries any particular question is $\frac{1}{2}$; and if he tries it, his chance of doing it is $\frac{1}{3}$. Find the odds in favour of any particular question being solved by neither A nor B .

186.

1. If $bz + cy = a$, $cx + az = b$, $ay + bx = c$, prove that

$$x^2 + y^2 + z^2 + 2xyz = 1.$$

2. Find a quadratic equation such that the sum of its roots is 5 and the sum of the cubes of the roots is 40.

3. If a , b , c , d are in continued proportion, prove that

$$bc(8a + 12b + 6c + d) = (2a + b)(2b + c)(2c + d).$$

4. Prove that the excess of the sum of the cubes of two consecutive integers over the sum of the integers is equal to the continued product of the two integers and their sum.

5. Express $\frac{4x^3 + 8}{x^4 - 1}$ in partial fractions.

6. If the remainder when $f(x)$ is divided by

$$(x - a)(x - b)(x - c)$$

is $p(x - b)(x - c) + q(x - c)(x - a) + r(x - a)(x - b)$,
 find p , q , r .

7. If s is the sum of a series of terms in geometrical progression, of which x is the first and y the last, prove that the common ratio is $\frac{s - x}{s - y}$.

8. Find the number of arrangements of three different letters which can be formed from the ten letters q to z , q when it occurs being always followed by u .

187.

1. Simplify $\frac{x+1}{x-1} + \frac{x-1}{x+1}$ when (1) $x = \sqrt{\frac{3}{2}}$, (2) $x = \sqrt{-1}$.
2. If x_1, y_1 and x_2, y_2 are the pairs of solutions of the equations $ax^2 + by^2 = 1$, $lx + my = 1$, prove that

$$x_1x_2 + y_1y_2 = \frac{a + b - l^2 - m^2}{am^2 + bl^2}.$$
3. a, b, c are in continued proportion and are the mean proportionals to x and y , y and z , z and w respectively, prove that $\frac{x}{y} = \frac{z}{w}$.
4. If $ax^2 + 2bx + c$ is a factor of $ax^3 + 3bx^2 + 3cx + d$, prove that the former is a perfect square and the latter a perfect cube. Prove also that in this case the cube root of the latter is $a^{\frac{1}{3}}\left(x + \frac{d}{c}\right)$.
5. Solve the equations :

$$xy + x + y = 5, \quad xz + x + z = 7, \quad yz + y + z = 11.$$
6. Sketch roughly the graph of the function $\frac{x^n + x}{x^n + 1}$ when n tends to increase without limit.
7. Find the equation whose roots each exceed by one the roots of the equation $x^5 - x^4 - 7x^3 + 2x^2 + 10x + 4 = 0$.
8. A cask is full of spirit and one-hundredth part of the contents is drawn off daily, the cask being filled up with water each time. After how many days will there be more water than spirit in the cask?

188.

1. If $x^{13} = 1$, but $x \neq 1$, and $y = x + x^5 + x^8 + x^{12}$, prove that $y^3 + y^2 = 4y - 1$.

2. If the roots of the equation $\frac{1}{x-b} - \frac{1}{x} + \frac{1}{a} = 0$ are equal, find the value of $\frac{a}{b}$.

3. If $\sum_{r=1}^{r=n} (n+1)(n+4) \equiv an(n+b)(n+c)$, find the values of a, b, c .

4. Solve the equations:

$$(1) \quad y + z = yz - 1, \quad z + x = zx - 2, \quad x + y = xy - 5.$$

$$(2) \quad \frac{1}{x} - \frac{1}{y} = \frac{1}{x+1} - \frac{1}{y+2} = \frac{1}{12}.$$

5. If x is small, an approximate value of

$$\frac{(1-x)^{-\frac{2}{3}} - (1+x)^{\frac{2}{3}}}{(1-x)^{-1} - (1+x)}$$

is $\frac{2}{3} - \frac{2}{9}x$. Calculate the error per cent. when x equals $\cdot 1$.

6. If the square of any number is divided by 3, prove that it cannot have a remainder 2. Also if the square of any odd integer is divided by 8, prove that the remainder must be 1.

7. If x is real, prove that $\frac{x^2 - 50x + 625}{x - 50}$ can have no positive value less than 100.

8. If $\frac{x-y}{y-z} = \frac{3}{2}$ and $y^2 = xz$, find the ratios $x:y:z$.

189.

1. Simplify $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{2} + \sqrt{2} + \sqrt{3}} - \frac{\sqrt{3} - \sqrt{2}}{\sqrt{2} - \sqrt{2} - \sqrt{3}}$.

2. Find the two values between which $\frac{x^2 + 2x + 3}{x^2 + 3x + 2}$ cannot lie for real values of x .

3. Solve the equations :

$$(1) \frac{x+y}{xy+1} = \frac{x-y}{xy-1} = 1.$$

$$(2) \sqrt{x+1} + \sqrt{y-2} = \sqrt{x-1} + \sqrt{y+2} = 3.$$

4. If a, b, c, d, e are in continued proportion, prove that $\frac{b^2(a-c)}{a^2(b+d)} = \frac{e(b-d)}{c(c+e)}$.

5. If $1200 < x < 2600$, find the sum of all integral values of x which are not divisible by 7.

6. Prove that the sum of 2^r terms of the series

$$1 + 2 + 2^2 + 2^3 + 2^4 + \dots$$

is equal to the product of r factors of the expression $(2+1)(2^2+1)(2^4+1)(2^8+1)\dots$.

7. If α, β are roots of $x^2 + px + q = 0$ and are also roots of $x^{2n} + p^n x^n + q^n = 0$, where n is an even integer, prove that $\frac{\alpha}{\beta}$ is a root of $x^n + 1 + (x+1)^n = 0$ if $\alpha^n \neq \beta^n$.

8. The duration of a railway journey varies directly as the distance and inversely as the velocity. The velocity varies directly as the square root of the quantity of coal used per mile and inversely as the number of carriages in the train. For a journey of 25 miles in half an hour with 18 carriages 10 cwt. of coal is required. How much coal will be consumed in a journey of 15 miles in 20 minutes with 20 carriages?

190.

1. If $a + b + c = 0$, prove that $(a^2 + b^2 + c^2)^2 = 2(a^4 + b^4 + c^4)$.
2. If a, b, c are in harmonical progression and are unequal, and if $a + b - x, a + c - x, b + c - x$ are in geometrical progression, prove that $4ac, bx, b^2$ are in arithmetical progression.
3. If $\frac{x}{a+b-c} = \frac{y}{a+b+c} = \frac{z}{3a+3b+c}$, prove that $x + 2y - z = 0$.
4. Solve the equations :
 - (1) $x^2y - xy^2 = 6, \quad x^3 - y^3 = 19.$
(Only real solutions required.)
 - (2) $x^2 + y^2 + z^2 = 931, \quad x + y + z = 49, \quad z^2 = xy.$
5. Find a symmetrical homogeneous function of a, b, c of the third degree which is 0 when $a=1, b=0, c=0$, and is 1 when $a=1, b=1, c=0$, and is 3 when $a=1, b=1, c=1$.
6. Find a pair of real values of x and y which satisfy the equation $(2 - \sqrt{-1})x + (3\sqrt{-1} + 1)y + 2 = 0$.
7. If $\frac{p}{x^2 - yz} = \frac{q}{y^2 - zx} = \frac{r}{z^2 - xy}$, find the ratio $x : y : z$.
8. A train, which travels 40 miles an hour, starts from A at 9 a.m. and is timed to reach B at 10.12 a.m. An excursion train, which should start at 6.30 a.m. and travel half as fast, starts from B for A 54 minutes late and is delayed 3 minutes in every 2 miles. At what time of day will the trains pass each other?

K. S. N. M. S. HIGH SCHOOL,

Bangalore.

PAPERS 191–200 include

*Miscellaneous series; induction; binomial theorem;
indeterminate equations.*

191.

1. Divide $x^7 - 729x$ by $x^2 - 3x + 9$, and express the quotient in factors.

2. Prove that the values of x, y which satisfy the equations $x(b - c) + by - c = 0$ and $y(c - a) - ax + c = 0$ will also satisfy $ax - by + a - b = 0$.

3. If the coefficient of x^r and x^{r+1} in the expansion of $(3x + 2)^{19}$ are equal, find the value of r .

4. Prove by induction that $\sum_{r=1}^{r=n} \frac{1}{2}r(r+1)$ is $\frac{n(n+1)(n+2)}{6}$.

5. Solve the equations:

$$(1) \quad \frac{y}{x} - \frac{x}{y} = \frac{x+3}{y+3} = \frac{x+y}{xy}.$$

$$(2) \quad x^2 + xy + y^2 = 7, \quad xy(x+y) = 6.$$

6. Prove that n terms of the series

$1 + x(1+x) + x^2(1+x+x^2) + x^3(1+x+x^2+x^3) + \dots$
equals $(1-x^n)(1-x^{n+1})/(1-x)(1-x^2)$.

7. What is the shape of the surface $z = f(x) + f(y)$, where $f(x)$ represents the greatest integer in x ?

8. Find in the scale of 7 a number of four digits which is halved if the last digit is put in front of the other three.

192.

1. If $a + d = 2c$ and $\frac{1}{a} + \frac{1}{d} = \frac{2}{b}$, prove that $\frac{a}{b} = \frac{c}{d}$.

2. If $4 = \frac{3}{x_1} = x_1 + \frac{3}{x_2} = x_2 + \frac{3}{x_3} = x_3 + \frac{3}{x_4} = \dots$

prove by induction or otherwise that $x_n = \frac{3^{n+1} - 3}{3^{n+1} - 1}$.

3. If $x^2 + y^2 + z^2 = 1$, $x + 2y + 3z = 3$, $3x + y + z = 3$, prove that $x - 8y + 5z = \pm 3$.

4. Find the coefficient of x^r in

$$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \dots + (x+2)^{n-1}.$$

5. Prove that (1) $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

$$(2) \sum_{r=1}^n (r^2 + 2r) = \frac{n(n+1)(2n+7)}{6}.$$

6. If ϵ is small, prove that one root of the equation

$$x^3 + x^2 - 12x = \epsilon$$

is approximately

$$-\frac{1}{12}\epsilon + \frac{1}{1728}\epsilon^2.$$

7. $OABCD$ are five points on a straight line such that OA, OB, OC, OD are the roots of the equation

$$ax^4 - bx^3 + dx - e = 0.$$

If AB, AC, AD are in harmonical progression, prove that $ad^2 = b^2e$.

8. The greater of two numbers each of two digits is three times the smaller, and the ten's digit of the latter is the same as the unit digit of the former. Also the sum of the digits of the greater is equal to the difference of the digits of the smaller. Find the numbers.

193.

1. If $a + b + c + d = 0$, prove that

$$(a^2 + b^2 + c^2 + d^2)^2 = 2(a^4 + b^4 + c^4 + d^4) + 8abcd.$$

2. Prove that $\sum_{r=1}^n (r+1)(r+4) = \frac{1}{3}n(n+4)(n+5)$.

3. In how many ways can 9 different things be distributed into sets of three?

4. If $(x+1)(y-1) = (y+1)(z-1) = (z+1)(x-1)$, prove that either $x=y=z$ or that each of these products equals -4 .

5. Solve the equations :

$$(1) \quad 3x^2 + y^2 = 13x^2y^2, \quad 3x + y = 7xy.$$

$$(2) \quad 2x^2 - (4 - \sqrt{-1})x + 5 - \sqrt{-1} = 0.$$

6. If $u_n = 2u_{n-1} + 1$, prove that $\frac{u_n + 1}{u_1 + 1} = 2^{n-1}$ and, if $u_1 = 1$, prove that $u_n = 2^n - 1$.

7. If the last digit of a perfect square is 6, prove that the digit next before it must be odd; prove also that if this digit is 7, the last digit but one in the square root must be either 7 or 2.

8. Find a relation between r and n if the coefficients of the $(2r+3)$ th and the $(3r+2)$ th terms of $(1+x)^{4n}$ are equal.

194.

1. Prove that $x^2 + 2x\sqrt{-1} + 1$ is a factor of

$$5x^4 + 12x^3\sqrt{-1} - 1.$$

2. Prove that the sum of the eleventh power of roots of the equation $x^5 + 5x + 1 = 0$ is zero.

3. Prove by induction that $(3n+1) \cdot 7^n - 1$ is divisible by 9, if n is a positive integer.

4. If $\frac{a+b}{b+c} = \frac{c+d}{d+a}$, prove that $a=c$ or $a+b+c+d=0$.

5. Express in partial fractions $\frac{a+x}{(a-x)^2(a^2+x^2)}$.

6. If α, β, γ are the roots of the equation $x^3 + ax + b = 0$, form the equation whose roots are

$$\beta^2 + \gamma^2 - \alpha^2, \quad \gamma^2 + \alpha^2 - \beta^2, \quad \alpha^2 + \beta^2 - \gamma^2.$$

7. Trace roughly the graph of $\frac{x^3}{x^2-1}$.

8. The chance that A adds up correctly a column of figures is p , and the chance that B does it correctly is q . They each add up the column. What is the chance that the correct result is obtained by one of them only?

195.

1. If $x+y+z=0$, prove that

$$x(y-z)^3 + y(z-x)^3 + z(x-y)^3 = 0.$$

2. Prove that $\sum_{r=1}^n (r^2+1) \mid \underline{r=n} \mid \underline{n+1}$.

3. If $s_1, s_2, \dots, s_m$ are the sums of arithmetical progressions, each of n terms, the first terms being 1, 2, 3, ..., m and the common differences 1, 3, 5, ..., $2m-1$, prove that

$$s_1 + s_2 + \dots + s_m = \frac{1}{2}mn(mn+1).$$

4. Solve the equations

$$(1) \quad 5\sqrt{\frac{3}{x}} + 7\sqrt{\frac{x}{3}} = 22\frac{2}{3}.$$

$$(2) \quad \frac{1}{c^2-x} + \frac{1}{d^2-x} + \frac{1}{2cd-x} = 0.$$

5. Prove that

$$1 + (2n+1) + \frac{(2n+1)2n}{\underline{2}} + \dots + \frac{(2n+1)2n(2n-1)\dots(n+2)}{\underline{n}} = 2^{2n}.$$

6. Prove that every cube is of the form $7n$ or $7n \pm 1$.

7. A graduated income tax is arranged so that a man whose income is x £ pays a tax of $100 \left(1 - \frac{a}{x+b}\right)$ per cent. of his income, where a, b are constants so chosen that an income of £1000 pays 1 shilling in the pound and an income of £10,000 pays 2 shillings in the pound. Find the greatest possible income a man can have after income tax has been deducted.

8. a, b, c are in arithmetical, x, y, z in harmonical and ax, by, cz in geometrical progression. Prove that either

$$ax = cz \quad \text{or} \quad az = cx.$$

196.

1. If $\frac{ad-bc}{a-b-c+d} = \frac{ac-bd}{a-b+c-d}$, prove that each fraction $= \frac{1}{4}(a+b+c+d)$, if $a \neq b$ and $c \neq d$.

2. If $s_1 = 0$ and $s_{m+1} = (1-x)s_m + mx$ for all positive integral values of m , prove that $s_m = \frac{1}{x}\{mx - 1 + (1-x)^m\}$.

3. Prove that

$$1 - n \frac{1+x}{1+nx} + {}^nC_2 \frac{1+2x}{(1+nx)^2} - {}^nC_3 \frac{1+3x}{(1+nx)^3} + \dots = 0.$$

4. Solve the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$, given that its roots are of the form $a+1, a-1, b+1, b-1$.

5. In any geometrical progression of an odd number of terms, prove that the sum of the squares of the terms equals the product of the sum of the terms by the excess of the sum of the odd terms over the sum of the even terms.

6. Solve the equations :

$$(1) \quad x^2 - xy + y^2 = x - y + xy = \frac{4}{6} \frac{9}{4}.$$

$$(2) \quad \frac{\sqrt{x}}{\sqrt{x+a} - \sqrt{a}} + \frac{\sqrt{x}}{\sqrt{x+a} + \sqrt{a}} = 1 + \frac{a}{x}.$$

7. Prove that the mid term of $(1+x)^{2n}$ is the same as the $(n+1)$ th term of $(1-4x)^{-\frac{1}{2}}$.

8. One of the terms of an arithmetical progression of integers is 0, the sum of a certain set of consecutive terms is 24 and of another set is -51. Prove that the common difference is ± 1 or ± 3 .

197.

1. Prove that $3x^2y^2z^2 = \Sigma(x^3y^3) + 3xyz \cdot \Sigma(x) \cdot \Sigma(xy) - [\Sigma(xy)]^3$.

2. The sum of a series of n terms in arithmetical progression is a , and the sum of their squares is b . Prove that the first and last terms are the roots of the equation

$$(n+1)(nx-a)^2 = 3(n-1)(nb-a^2).$$

3. Prove that $3^{2n+4} - 2^{2n}$ is divisible by 5.

4. If $q-p$ is small, prove that the n th root of $\frac{p}{q}$ is nearly equal to $1 - \frac{2}{n} \cdot \frac{q-p}{q+p}$.

5. Solve the equations :

$$(1) \quad 4y = 4x^3 - 17x, \quad 4x = 4y^3 - 17y.$$

$$(2) \quad x(y+z) = 27, \quad y(z+x) = 20, \quad z(x+y) = 35.$$

6. Prove by induction that

$$1 - 2^4 + 3^4 - 4^4 + \dots + (-1)^{n-1}n^4 = (-1)^{n-1} \frac{1}{2}n(n^3 + 2n^2 - 1).$$

7. If $\frac{b}{a+b} = \frac{a-b+c}{b+c-a} = \frac{a+b+c}{2a+b+c}$, find the ratios $a:b:c$.

196 TEST PAPERS IN ELEMENTARY ALGEBRA

8. A farmer spends £184 in buying sheep at 2 guineas a head and oxen at £12. 10s. a head. How many of each did he buy?

198.

1. If $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = a^3 + b^3 + c^3 = 0$, prove that

$$(a + b + c)^3 \left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} \right) = -9.$$

2. The arithmetic mean of a and b is u_1 , that of b and u_1 is a_2 , that of u_1 and u_2 is u_3 and so on. Prove that u_n equals $\frac{1}{3} \left[\left\{ 1 - \left(-\frac{1}{2} \right)^n \right\} a + \left\{ 2 + \left(-\frac{1}{2} \right)^n \right\} b \right]$.

3. If a is so nearly equal to b that $(a - b)^2$ may be neglected, prove that $\frac{2a + b}{a + 2b}$ may be taken as the cube root of $\frac{a}{b}$.

4. Find the number of combinations of $p + q$ things taken r at a time when p are of one set and q of another and $r > p > q$.

5. Prove that $\sum_{r=1}^n \frac{1}{2} r^5 (1 + r^2) = \left(\frac{n(n+1)}{2} \right)^4$.

6. Solve the equations:

$$(1) \quad x^2 + y^2 + z^2 = 931, \quad x + y + z = 49, \quad z^2 = xy.$$

$$(2) \quad \sqrt{x+7} + \sqrt{2x-9} - \sqrt{5x+4} = 0.$$

7. Trace roughly the graph of $y^2 = x(x^2 - 1)$.

8. If $x^3 + 2px^2 + 2qx + r$ and $x^2 + px + q$ have a common factor of the first degree in x , prove that $r^2 - 3pqr + p^3r + q^3 = 0$.

199.

1. Prove that

$$\begin{aligned} & \frac{(x-b)(x-c)(x-d)}{(a-b)(a-c)(a-d)} + \frac{(x-a)(x-c)(x-d)}{(b-a)(b-c)(b-d)} \\ & + \frac{(x-a)(x-b)(x-d)}{(c-a)(c-b)(c-d)} + \frac{(x-a)(x-b)(x-c)}{(d-a)(d-b)(d-c)} = 1. \end{aligned}$$

2. Express $\frac{2x^2 - 11x + 11}{(1-x)(x-2)^2}$ in partial fractions, and show that if it can be expanded in ascending powers of x , it equals

$$\sum \left[2 + \frac{3(n+1)}{2^{n+2}} \right] x^n.$$

3. Prove that the n th terms in $(1-x)^{-n}$ and $(1+x)^{2n-2}$ are equal.

4. $\alpha \pm \beta$ are the mid pair of the n terms of an arithmetical progression, n being even. Prove that the sum of the cubes of the terms is $n\alpha\{\alpha^2 + (n^2 - 1)\beta^2\}$.

5. Simplify $\sqrt{\frac{1}{2} - \frac{\sqrt{2}}{3}}$.

6. Solve the equations :

$$(1) \frac{x^2}{4} + \frac{y^2}{9} = 2, \quad \frac{4}{x^2} + \frac{9}{y^2} = 2.$$

$$(2) \frac{a}{x^2 - cx + b} - \frac{b}{x^2 - cx + a} = \frac{a}{b} - \frac{b}{a}.$$

7. Prove that the number of ways in which n things may be arranged in a row so that 3 particular things are separated is $(n-2)(n-3)(n-4) \cdot \underline{n-3}$.

8. Prove that $\sum_{r=1}^n (r^3 + 3r^5) = 4 \left(\sum_{r=1}^n r \right)^3$.

200.

1. If $x = 1 + \frac{1}{y}$, $y = 2 + \frac{1}{z}$, $z = 3 + \frac{1}{x}$, prove that $xyz - \frac{1}{xyz} = 12$.

2. Solve the equations :

$$(1) x + y + z = ab, \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{b}{a}, \quad xyz = a^3.$$

$$(2) \sqrt[2m+1]{(1+x)^2} - 2 \cdot \sqrt[2m+1]{(1-x)^2} = \sqrt[2m+1]{1-x^2}.$$

198 TEST PAPERS IN ELEMENTARY ALGEBRA

3. If $f(x)$, $F(x)$ be quadratic algebraic rational expressions and if $\frac{f(x)}{F(x)}$ be expanded in powers of x , prove that no term can vanish unless $\frac{\log f(\alpha) - \log f(\beta)}{\log \alpha - \log \beta}$ is a positive integer, where α , β are the roots of $F(x) = 0$.

4. If $\frac{a+cx}{c+ax}$ is expanded in a series (1) in powers of $(1-x)$ and (2) in powers of $(1+x)$, and if A , B are the coefficients of $(1-x)^n$ and $(1+x)^n$ respectively, prove that

$$A(a+c)^{n+1} + B(a-c)^{n+1} = 0.$$

5. Prove that $\frac{4}{2 \cdot 3 \cdot 4} + \frac{7}{3 \cdot 4 \cdot 5} + \frac{10}{4 \cdot 5 \cdot 6} + \dots$ to n terms equals

$$\frac{5}{6} - \frac{3n+5}{(n+2)(n+3)}.$$

6. If x is small, prove that

$$\frac{\sqrt{1+x} + (1-x)^{\frac{2}{3}}}{1+x+\sqrt{1+x}} \text{ equals approximately } 1 - \frac{5x}{6}.$$

7. The quantity of water discharged per minute by an intermittent stream during each hour is constant, but at the beginning of each hour varies as the number of hours from the previous midnight and also as the number of hours from the succeeding midnight; the maximum number of gallons discharged per minute is 144. Find the whole quantity discharged per day.

8. If α , β are the roots of the equation $\frac{a}{c+x} + \frac{b}{d+x} = 1$, prove that

$$\left\{ a \frac{\alpha+d}{\alpha+c} + b \frac{\alpha+c}{\alpha+d} \right\} \left\{ a \frac{\beta+d}{\beta+c} + b \frac{\beta+c}{\beta+d} \right\} + (\alpha-\beta)^2 = 0.$$

GRAPHICAL EXAMPLES.

1. Mark on squared paper the points $(-1, 2)$; $(3, 1)$; $(1, 6)$; and find the area of the triangle formed by joining them.

2. Find the area of the quadrilateral whose vertices are $(-2, -3)$; $(3, -1)$; $(2, 6)$; $(-1, 4)$.

3. Draw the graphs of the functions (1) $3x - 2$; (2) $4 - x$. Hence solve the equations $3x - y = 2$, $x + y = 4$.

4. Solve graphically the equations $2x + 3y = 7$, $5x - 4y = 11$.

5. Draw the graph of the function $\frac{1}{5}(x+2)(x-1)$ between $x = -4$ and $x = +3$; and determine from the figure the values of x for which the function is equal to 1.

6. Draw the graph of $2x^2 - 3x + 1$, and use it to solve the equations :

$$(1) 2x^2 - 3x + 1 = 0. \quad (2) 2x^2 - 3x - 1 = 0. \quad (3) 4x^2 - 6x + 1 = 0.$$

7. Solve graphically the equations :

$$y = x^2 - x - 2, \quad 2x - 3y = 3.$$

8. Find graphically the minimum value of $2x^2 - 3x + 1$.

9. Find a point on the graph of the function $\frac{1}{4}(x^2 - 1)$ such that its x -coordinate is three times its y -coordinate. Measure the y -coordinate of the point.

10. Find by measurement the gradient of the function $\frac{(x-1)(x-2)}{5}$ when x equals 3, and determine approximately the value of x for which the gradient is 1.

11. Draw the graph of $x^2 + 3x$ between $x = -4$ and $x = +4$; and solve graphically $x^2 + 3x = 4$.

12. Draw the graph of the function $3 + 6x - x^2$, and hence find its greatest value.

200 TEST PAPERS IN ELEMENTARY ALGEBRA

13. Draw the graph of $y = x^2 - 8x$ and use it to solve $x^2 - 8x + 3 = 0$, and find the greatest value of c for which the roots of $x^2 - 8x + c = 0$ are real.

14. Find the gradient at the point $x = 3$, $y = 1.8$ on the graph of $y = \frac{1}{5}x^2$.

15. The vertices of a quadrilateral $ABCD$ are $A(0, 0)$; $B(2.4, 3.2)$; $C(2.4, 4.5)$; $D(0, 5.2)$. Find the lengths of AB and CD , and the area of $ABCD$.

16. It is ten miles from A to B . Two cyclists start from A and B respectively to ride the distance, the one from A starting 10 minutes before the other, who does the journey to A in 40 minutes. On arriving at B the first returns at once, and arrives at A at the same time as the other. Find his speed and the place where they first meet.

17. Draw with the same axes the graphs of $y = 2x^2$ and $y = x + 1$. What quadratic can be solved from the figure, and what are its roots?

18. Draw the graphs of $\frac{x^2}{10}$ and $2 + \frac{1}{x}$; hence solve the cubic $x^3 - 20x - 10 = 0$.

19. Construct the graph of a function y of x from the following table :

$x = 1$	2	3	5	6	7	8
$y = 1.30$	2.12	2.77	3.71	4.05	4.32	4.52

and estimate the values of y when $x = 4$ and $x = 9$.

20. A thief steals a motor car at 7 p.m. and drives off at 25 miles an hour, followed ten minutes later by the owner, who travels 24 miles an hour while driving, but spends 15 per cent. of his time in making enquiries. The thief, after driving 15 miles, transfers the car to an accomplice, who drives 6 miles

at 36 miles an hour and then breaks down. After spending 15 minutes in repairs he continues at 16 miles an hour. When will he be overtaken?

21. Three men, with only two bicycles, want to go a journey of 12 miles. They can each bicycle 12 miles an hour and walk 4 miles an hour. After a man has ridden a certain distance he leaves the bicycle on the roadside for his friend to pick up and walks on. How must they arrange matters if they are to start together and arrive together in the shortest time possible? How long will they take? Will the two bicycles ever be lying on the roadside at the same time?

22. Solve graphically $3y - 4x = 1$, $y^2 - 3x = 3$.

23. Find the area of the pentagon whose vertices are $(2, 8)$; $(-1, -1)$; $(6, 1)$; $(0, -4)$; $(-5, 1)$.

24. Draw on the same axes the graphs of
 $y = x - 3$, $y + 2x = 0$, $y = x^2 - 6x + 3$.

Hence solve the equations:

(1) $x^2 - 6x + 4 = 0$. (2) $x^2 - 4x + 3 = 0$. (3) $x^2 - 7x + 6 = 0$.

What is the greatest value of $(x - 3) - (x^2 - 6x + 3)$?

25. A train travelling 50 miles an hour passes four stations A , B , C , D at 3.0, 3.30, 4.10, 4.25. A second train starts from D at 3.20 and travels in the opposite direction for the first half hour at 30 miles an hour, and afterwards at 60 miles an hour. At what times does it pass C , B and A ? When are the two trains together, and when are they 30 miles apart?

26. X , Y are two towns 100 miles apart. A leaves X at 10 a.m. and travels towards Y at 10 miles an hour; B leaves Y at 10.30 a.m. and travels towards X at 12 miles an hour. At the end of each hour's riding, A rests for 5 minutes and B rests for 10 minutes. How far from X do they meet?

202 TEST PAPERS IN ELEMENTARY ALGEBRA

27. Draw the graphs of $y = x^2 - x$ and $y = \frac{4}{x^2}$ between the values $x = -4$ and $x = 4$, and show that the equation $x^4 - x^3 = 4$ has only two real roots.

28. A, B, C, D are four stations on a railway ; $AB = 5$ miles, $BC = 10$ miles, $CD = 5$ miles ; a railway time table reads as follows :

Up.	Down.
A dep. 10.5 a.m.	D dep. 10.10 a.m.
B arr. 10.16 a.m. dep. 10.19 a.m.	C —
C arr. 10.35 a.m. dep. 10.37 a.m.	B —
D arr. 10.48 a.m.	A arr. 10.40 a.m.

Where do the trains pass each other ?

29. Draw the graph of $2x + 3y = 20$, and from it write down all the positive integral solutions of the equation.

30. Draw the graphs of $y = x^2$ and $xy = 7x - 2$ between $x = -5$ and $x = 5$. Hence solve the cubic $x^3 - 7x + 2 = 0$.

31. Determine graphically the values of x between which $41 - 4x - 4x^2$ is positive ?

32. The speed of an electric tram is reduced in ten seconds by the brakes in the following way :

Time in seconds	0	1	2	3	4	5
Speed in miles per hour	30	27.3	25	23.1	21.4	20
Time in seconds	6	7	8	9	10	
Speed in miles per hour	18.8	17.6	16.7	15.8	15	

Show that the tram travels about 305 feet before coming to rest.

33. Soundings are taken at intervals of four feet across a river 40 feet wide, starting 4 feet from one bank, and the following depths (in feet) are obtained in order: 6·6, 9·3, 9·9, 8·2, 8·4, 10·2, 10·5, 7·8, 4·5. Obtain from a figure the approximate area of the cross section of the river at this place.

34. Describe the variation of the function $\frac{x-1}{x-2}$ as x passes from large negative to large positive values.

35. The population of a certain state is numbered every 5 years:

Date	1875	1880	1885	1890	1895	1900	1905	1910
Population in millions	6·00	9·02	11·44	13·23	14·39	14·97	14·90	14·23

Show this graphically, and estimate the population in 1883 and the annual rate of increase in 1887.

36. A man five feet eight inches in height walks towards an electric arc street light at the rate of 4 miles an hour. The light is 25 feet above the level of the street. Draw a graph showing the relation between the length of his shadow and the distance from the foot of the lamp.

37. The following table gives the results of a series of experiments made to determine the relation between two quantities x and y :

$x = 36·8$	31·5	26·3	21	15·8	12·6	8·4
$y = 12·5$	12·9	13·1	13·3	14·1	14·5	16·3

Show graphically the relation between x and y , and also

between x and xy . From the second graph obtain the probable algebraic relation between x and y .

38. The following table shows the pressure (p) in lbs. p. sq. in. and the volume (v) in cu. ft. of 1 lb. of saturated steam

$p = 12$	14	15	17	20	22	25	30
$v = 32.0$	27.7	26.0	23.1	19.8	18.1	16.1	13.5

Show graphically the relation between p and v , and find the value of p for which $\frac{v}{10} + \frac{p}{11} = 4$.

39. A straight line KL is 12 cms. long. At the ends of this straight line and on the same side of it are erected perpendiculars KA 6 cms. long and LB 3 cms. long. P is a point on KL such that $KP = x$ cms. Show graphically how $AP^2 + PB^2$ varies with x , and determine x when

$$AP^2 + BP^2 = AB^2.$$

40. AB is a straight line of length 10 cms. ; P is any point on the semicircle on AB as diameter ; N is the foot of the perpendicular from P to AB . By taking a number of positions of P and measuring AP , AN , show the relation between AP and AN when N moves from A to B , and determine from the graph the length of AN when $AP = 2AN$.

ANSWERS.

1.

3. $0, 2\frac{1}{6}$. 5. $29a - 23$. 6. $x - 3z$. 7. $8a + 3b$.

2.

1. 61. 6. $-9a + b + ab + 10, 5 - 5a$.

3.

1. $-4, -4, -44$. 6. $-35, 9xy^2 - 9x^2y + 3x^2 - 3y^2$. 7. $2x - 2y - z$.

4.

3. 0, 2. 5. $47x + 34, 7a - b + c, -14x^2$.
6. 71, $-12, 2$. 7. $16x^2 - 3x - 1$. 8. $2x + 1, 13, 28$.

5.

1. $2, \frac{1}{2}, 0$. 2. $5c - 2a - 3b$. 4. $-6, -1$.
5. $-\frac{a}{4} - \frac{7b}{4}, -2a, x^2(4y^2 + z^2 - 1) - 7y^2z^2$. 6. 7, 10; $24x + 4y; -225$.
7. $2x^2 - 4xy + 4y^2, x^5 - 6x^4 + 2x^3 + 22x^2 - 7x - 12$. 8. 270.

6.

3. 6. 4. $6x^3 - x^2 - 13x - 3$.
6. $-19a - 18b + 39c, 6x^2 + 3y^2 - z^2 - 4xy + 7yz - zx, 5$. 7. 7, 3, $1\frac{3}{11}$.

7.

1. $\frac{2}{3}, 3\frac{1}{6}, \frac{1}{20}, -4\frac{1}{2}$. 3. $-12x + 7y + 15z, -3$.
5. $a(a + 2b + 2c)$. 6. $-2a + 2b - 3c, 2a^2 - 2a - 3b + 6$.

206 TEST PAPERS IN ELEMENTARY ALGEBRA

8.

1. 3, 4, $-\frac{3}{8}$, 9.
2. $3x^3 + 4x^2 - x - 1$.
4. $7x^3 + 8x^2 - 5x - 6$, $2x^2 - 9xy - 7y^2$.
5. $6a + 24b + 27$, $\frac{2a}{3} - \frac{7b}{12}$.
8. $\frac{1}{6}(26a - 15b + 11c)$, $\frac{1}{2}(3a - 9b - 12c)$, 1.

9.

1. 9, 24, 10, 0, 5.
2. $3a - b - c - 4d$, $\frac{7x}{4} + \frac{y}{2} - \frac{5z}{12}$.
3. $\frac{11x}{2}$, $\frac{11x}{2} - 30$ degrees.
4. $-3a$, $-a - b$, $-8x^2 + 4xy + 4y$.
5. $7x^7 - 14x^6 + 19x^5 - 11x^4 - 2x^3 - 3x^2 + 8x - 4$.
8. $\frac{22x}{15}$.

10.

2. 45, 9, 0.
3. -53 , $24a^4b^8$, $2a - b$.
5. $a + 6b - 9c - 9d$.
6. x^2 .
8. -6 .

11.

1. 36, 0, 0, -64 .
3. 13.
4. 29.
5. $-81x^2 + 54xy$, $5ab + 7bc - 4ca$.
6. 0, 8.
7. $a^3 + b^3 + c^3 - 3abc$.
8. 8, 10, 12.

12.

1. 4, 16, -12 .
3. -2 .
4. 150.
5. $-\frac{1}{4}$, $\frac{4-5x}{12}$.
6. $CD = 5(x \sim y)$.
7. 8.

13.

1. $10x^2y + 55xy^2$.
2. $5ab - 5bc + 2ca$.
3. $860x - 868y + 964z$.
4. 3, 48, 2.
6. 154, $3\frac{9}{11}$.
7. 28, 14.
8. -2 , 4.

14.

1. $7a - b - 5c$, 10.
2. $2\frac{3}{4}$.
3. $\frac{16a}{15}$, $\frac{12a + 11b}{20}$, $\frac{15}{14ab}$.
4. 3, 6, 2.
5. $\frac{88x}{3m}$, $\frac{60my}{x}$.
6. 68£.
7. $\frac{7x}{3}$.
8. 4, 4.

15.

- | | | |
|---------------|----------------|----------------------------------|
| 1. $6x - 1$. | 2. $3c - 2b$. | 3. 12, -8, $-\frac{7}{13}$, 12. |
| 5. 11, 3. | 7. 60. | 8. -11. |

16.

- | | | |
|--------------------------------|---|------------------|
| 1. 20, 42, 34, $\frac{3}{5}$. | 2. $-\frac{1}{12}(x+18)$, $\frac{1}{30}(11x+18y-34)$, $\frac{9+20x}{10x}$. | |
| 3. 41, 43. | 4. $\frac{3}{7}$, $4\frac{1}{3}$. | 5. $557 - 27x$. |
| 6. 184, 336, 456, 56. | 7. 42. | 8. 12 miles. |

17.

- | | | |
|--|---|-------|
| 1. $-\frac{2}{3}$, $\frac{8}{3}$, 3. | 2. $\frac{7y^2 - 10x^2}{12xy}$, $-9x^2 + 25x - 8$, $\frac{27x - 23y}{30}$. | |
| 3. $5y$, $\frac{2x - 3y}{2}$. | 5. $\frac{3}{19}$, 60. | 6. 3. |
| 7. $\frac{x - 2y + 7z}{2}$. | 8. 12£. | |

18.

- | | | | | |
|---|-----------------------------|----------------------------|--------|-------|
| 1. $5ax + 3ay - 2xy - 3ab$, $-15a + 11b + 13c$. | 2. 1, 1, 1. | | | |
| 3. 8 inches. | 4. $\frac{1}{6}(3x + 4y)$. | 5. 20, 3, $1\frac{1}{2}$. | 6. 25. | 7. 7. |

19.

- | | | | |
|--|--------------------------|--------|------------------------|
| 1. $-\frac{ab}{6} - \frac{23bc}{36} + \frac{11ca}{8}$, $\frac{1}{30}(13a - 67)$, $-\frac{11x}{2y}$. | | | |
| 2. 55, 155, 15150, $n(2n+1)$, $\frac{n}{2}(2k+n+1)$. | 3. 10, $16\frac{1}{2}$. | | |
| 4. $-4\frac{1}{7}$, 4, $-\frac{1}{5}$. | 5. 3 years. | 6. 18. | 7. $15\frac{10}{11}$. |

20.

- | | | |
|--|--|---------|
| 1. $\frac{3}{2}$, $-\frac{3}{2}$. | 2. $4a^2 - 4ab$, $a^2 + 9ab - 4b^2$. | |
| 3. $\frac{3y^2 - x^2}{xy}$, $\frac{1}{36y}(11x - 66y - 34)$. | 4. $6\frac{1}{4}$ feet. | 5. 45£. |
| 6. 4, $11\frac{2}{7}$, $1\frac{1}{3}$. | 7. $11\frac{1}{2}$ feet. | |

208 TEST PAPERS IN ELEMENTARY ALGEBRA

21.

1. $12x^5 - 14x^4 - 3x^3 + 7x^2 - x - 1$, $3x^3 + 2x^2 - 3x - 2$.
2. -4 , -2 , 0 , 2 , 4 , 6 .
3. $\frac{5}{4}(11x + 3y - 4z)$, $\frac{5}{4}(5x - z)$.
4. $-5\cdot6$, $-\frac{1}{2}$, $\frac{1}{3}(a + 2b)$.
5. 12 .
6. 28 miles.

22.

1. $4 + 3x - x^2$, $x^2 - 2x + 3$.
2. $\frac{2(b-a)}{bx}$.
3. $3\frac{3}{7}$ m.p.h.
4. 1 ; -17 ; 2 , 3 .
5. A 30s., B 24s.
6. 70 scholars, 380 commoners.

23.

1. $2x + 2y - 4$ sq. ft.
2. $-\frac{1}{2}$; -52 ; 4 , -3 .
3. 200, 300, 100.
4. $-.75$, $5\cdot74$ or $-1\cdot74$.
5. 300£.

24.

1. $-3x^2 + 5xy + y^2$, $3\frac{4}{15}$.
2. $3x^2 - 11xy + 10y^2$ £.
3. $10\cdot4$; 11 ; 2 , 3 .
4. 24, 96, 60 degrees.
5. 7.

25.

1. $x^2 - 5x + 6$.
2. $x + 1$, $2 - 2x^3$, $\frac{9x^3 + x^2y - 8xy^2 + 12y^3}{6x^2y^2}$.
3. 480£.
4. 3, 1.
5. $-\frac{5}{11}$; $\frac{1}{5}$, $\frac{1}{3}$.
6. A 18s., B 16s.

26.

1. $\frac{1}{6}(3z - 3x - 8y)$.
2. $\frac{2y+5}{9y-2}$.
3. 4, 7, 18.
4. $1\frac{1}{6}$; 1 , $-\frac{1}{5}$.
5. $-2\cdot25$, $4\cdot56$ or $\cdot44$.
6. $29\frac{1}{3}$ miles.

27.

1. $x^8 - x^6 + 2x^2 - 2$, abc .
2. 144 ft., 144 ft., 2 secs.
3. -1 , 3 .
4. 2, 5.
5. 2, 3; $1\cdot5$ or -3 .
6. $\frac{8}{15}$.

28.

1. $-.7$, $x + 2a$.
2. $\frac{18x^2 + x + 5}{30x}$, $1 + \frac{1}{x^3}$.
3. 8, 1.
4. $3\cdot08$, 22.
5. $\pm 3\cdot74$.
6. 32.

29.

1. $x^4 - 62x^2 - 128$, $x^6 - y^6$. 2. 11; 5, 2. 4. 21. 5. 2, $-\frac{1}{6}$.
 6. $-10x - 34$, $\frac{4(c-b)}{bc}$. 7. $\frac{3xy}{88}$. 8. $3\frac{1}{9}$ miles.

30.

1. $2x^3 + x^2 - 3x - 7 - \frac{11}{x} - \frac{6}{x^2}$, $1 + 2x + 3x^2 + 4x^3 + 5x^4$.
 4. $\frac{14x-9}{21}$. 5. 4; 10, 12. 6. 792. 7. 20.

31.

1. $2\frac{2}{5}$, 3. 2. $\frac{x^2 - 4xy + 2y^2}{xy}$, $\frac{8ab + 2ac - a^2 + b^2 - c^2}{abc}$.
 4. $\frac{7a-6b}{40}$. 5. -1, $-\frac{11}{9}$. 6. 6 miles uphill.

32.

1. $2ax + cy$, $\frac{1}{12}(38a - 15b - 17)$. 2. $x^2 - 3xy + y^2$, -7, 10.
 4. $y - \frac{xz}{1440} - \frac{xy}{86400}$ minutes past z. 5. 30 miles.
 6. $\frac{1}{2}$. 7. $\frac{5}{7}$, $\frac{1}{7}$. 8. 3 hrs. 24 min.

33.

1. $x^3 + 4x^2 - 8x - 2$, $a^4 + 4a^2bc - b^4 + 2b^2c^2 - c^4$.
 3. $\frac{29x-2y}{30x}$, $a^2 + b^2 + c^2$. 5. $\frac{5}{6}$; $\frac{1}{2}$, $\frac{1}{5}$, $-\frac{1}{2}$.
 6. 63, 64 miles. 7. $5\sqrt{x^2 - \frac{6}{5}x + 1}$. 8. 3.70 or -2.70.

34.

1. $q^2 - y^2$. 2. $\frac{19}{47}$; $\frac{1}{6}$; 7, -3. 4. $8\frac{3}{4}$ hrs.
 5. $\frac{47}{29} = 1.62$, $\frac{16}{29} = .55$. 6. $2x^2 - x - 1$.
 7. 13 m. from A. 8. 8, $6\frac{1}{2}$.

210 TEST PAPERS IN ELEMENTARY ALGEBRA

35.

1. $x(a+b) - y(a-b) - 2ab$.
2. $12x - 1, \frac{17}{6} + \frac{2y}{3x}$.
3. 385, 6985, $\frac{a}{6}(a+1)(14a+1)$.
5. $\frac{2x}{5} - 6$.
6. $11; \frac{7}{4}; 5\frac{1}{4}, -\frac{3}{4}$.
7. 40 ft.
8. 7 boys.

36.

2. $-x^2 - x - 1, \frac{5}{3}(x-3y)$.
3. $x^2 - xy + y^2, 15a^3 - 26a^2b - 13ab^2 + 28b^3$.
4. $\frac{13x}{20}, \frac{3x}{20}$.
7. $5; -\frac{7}{8}, -\frac{1}{4}$.
8. 4.30 p.m.

37.

1. $\frac{1}{2}x^3 - 1, x^3 + 3x^2 + 9x$.
2. $\frac{1}{3}(2y-x)$.
3. $3x - \frac{2}{12}, \frac{y}{x}$.
4. 3, 3, $4 > x > 2\frac{4}{7}$.
6. $-1.8; -2, 1$.
7. 16.
8. 39.

38.

2. $-1\frac{1}{6}$.
3. $22qx = 15py$.
5. $\frac{1}{2}(x^2 - x - 6)$.
6. 3 or -1 .
7. 9 men.
8. 20.

39.

1. 2025, 42075, $9(a+4)(a^2+8a+36)$.
3. 42.
5. $x+y+1$.
6. $\frac{2}{5}; \frac{2}{3}, -5\frac{2}{3}$.
7. Coffee 1s. 9d., tea 2s. 4d.
8. 20 miles from X

40.

1. 69, 127.
2. $x^4 - x^3 - 19x^2 - 11x + 30, x^2 + 5x + 6, -\frac{1}{11}$.
3. $-\frac{42x+59y}{6y}, \frac{20a^4b^4}{21c^2}$.
5. $1056\left(\frac{x}{y} - \frac{p}{q}\right)$.
6. $-\frac{1}{3}; 2.8, .4$.
7. $\frac{17}{24}$.
8. 20 and 35, or 15 and 40 miles.

41.

1. 0.
2. 17, 15.
3. $x - y + 8z$.
5. $\left(\frac{x+3}{x+1}\right)^2, \frac{1}{30(x+3)}, -\frac{1}{x+3}$.
6. 7 years.
8. 13.

42.

1. 2 or -1. 4. $-2\frac{2}{3}$; 3, -1, 1. 5. $\frac{4}{2}\frac{5}{2}$.
 6. 30£, 12£, 6£. 8. $\frac{2}{3}$ ton, 1 ton.

43.

2. $-\frac{a}{b}$, 1. 3. $\frac{2xy}{y^2-x^2}$. 5. $\frac{1}{5}$; 2, 4, -2.
 7. $8\frac{2}{5}$ min. to 12. 8. 21, 42, 63, 84.

44.

2. $\frac{14x}{15(2-x)}, \frac{x+2}{x-5}$. 3. $\frac{bcz}{xy}$. 4. $(x-2)(x-3)(x+5)$.
 5. 2£. 1s. 6. ± 2 ; 9, 7. 8. 25£.

45.

1. $-1\frac{1}{2}$, 9, 13, 45. 2. -8. 3. 14.
 5. $\frac{x(x+3)}{(x+1)(x+2)}, \frac{x-3}{x+3}, \frac{1}{x+y}$. 6. $2x-y=9$, 4£.
 7. 6, 9; 2, -1, 1. 8. 5s., 2s. 6d., 1s. 6d.

46.

1. $\frac{y}{60x}, \frac{88a}{bx}$. 2. $-\frac{a+7b}{7a+9b}, -\frac{a^2+b^2}{3a^2+2ab-3b^2}$. 3. 84, 85.
 4. $15(x^2+x)(x^2-4)$. 5. 3; 4, 1. 6. $x+y+\frac{py}{q}$. 8. 2£.

47.

1. 11, x^2+2x+1 . 3. $py+(p-1)x$. 5. 25.
 6. $\frac{x-5}{x+3}, -\frac{9}{8}x^{10}y^9z^6, -2$. 7. 320, 252. 8. 80£, 65£.

48.

2. -3. 3. 315. 4. 13; 7 or -1; -1, 2, 3.
 5. $\frac{xy(a+b)}{bx+ay}$. 6. 1, $\frac{1}{a}$. 8. 4s. 6d.

49.

1. $30y \sim \frac{11x}{2}, 5\frac{5}{11}$ or $38\frac{2}{11}$ min. past 4. 3. 1, 1, 1.
 5. 25£, 16£, 18£. 6. $-3\frac{2}{3}$; 20, -10.

212 TEST PAPERS IN ELEMENTARY ALGEBRA

50.

3. $2a - 2b, \frac{3}{(x+1)(x+2)}$.

4. 6.

6. $2\frac{3}{4}$.

7. 66 yards.

8. $\frac{1}{6}\frac{6}{3}$.

51.

1. $x^3 + x^2 + 2x + 3$. 2. $\frac{2(b^2c^2 + a^2d^2)}{abcd}, x + \frac{20}{x} - 9, \frac{1}{(x-1)(x+2)}$.

3. 6.

4. 2 or 6; $\frac{5}{4}, \frac{1}{4}$.

7. 36£, 34£, 18£.

8. 104.

52.

1. $\cdot 9949875 > \cdot 9949874$. 2. $x = z, 1$. 4. $2(x^2 + y^2), 2(x^2 + xy - y^2)$.

5. $-\frac{1}{5}$; 1 or 5; 16, $-12\frac{1}{2}$. 6. 200, 150. 7. 1809 A.D. 8. 100.

53.

1. $x^2 - 12x + 35$.

4. 2s., 1s. 8d.

5. -10; 6 or -1; 3 or -5, 3 or 11.

8. 21, 28, 31.

54.

2. $\frac{2x}{x-y}, 1$. 3. $x^2 + 2x + 1, 3$.

4. $\frac{21}{50}$.

6. -4; 0 or 5; 3, -1, 7.

7. 10, 12 miles.

55.

1. $5xy$. 4. $1 + \frac{1}{x}, \frac{x+1}{x^2+1}$.

5. 6.

6. $1\frac{1}{5}$; -89, 8, 260; 5 or $-\frac{7}{15}$.

7. 9.

8. 12.

56.

1. $7x - 1$. 2. 1, $x^2 + y^2$. 3. 9. 5. 7; 4 or -2; $\pm 3, \pm 7$.

6. 33 miles.

7. $-5\frac{1}{6}$.

8. 12.

57.

1. 37. 2. $2, \frac{abc(a+b+c)}{(b-c)(c-a)(a-b)}$.

4. 6, 13.

5. 7, 2; 3, 2, 1; 1 or -3.

6. 32 miles.

8. 25.

58.

1. 0. 2. 2. 4. $\frac{24000}{100x+xy}$ 5. $\frac{a-1}{2a+2}, \frac{x^3-y^3}{x+y}$.
 6. £1200. 7. $-1\frac{3}{10}, 3\frac{1}{2}; 2 \text{ or } \frac{1}{2}$. 8. 1760, 1056.

59.

1. $5x^2-x+3$. 2. 74^2+92^2 . 3. 49.
 4. $\frac{b^2}{a^2} - \frac{a^2}{b^2}, \frac{(a+b-c)(b+c-a)(c+a-b)}{(a+b+c)^3}$.
 5. $4\frac{1}{5}; 2a-b, a+3b; 10$. 6. 1440. 7. 14 ft. 8. 15 miles.

60.

1. Each is -9. 2. $\frac{1}{ab}$. 4. $4y(x-y)$.
 5. $\frac{5}{7}; \pm 2; 2, 3, 4$. 6. 54. 8. £2000.

61.

1. $1, \frac{1}{2}\sqrt{7}$. 3. $-1, 1, 5, \frac{3}{2}$. 4. $x+4y=3z$.
 6. $-\frac{1}{3}; \frac{-5 \pm \sqrt{17}}{2}; 1.06$. 7. 280. 8. 4s.

62.

2. 1. 4. $\frac{(a-b)^2}{b}, 1$. 6. $4; 3, \frac{1}{4}; \frac{7 \pm \sqrt{57}}{4}$.
 7. 12, 21 m.p.h. 8. 4.

63.

3. 4. 4. $\frac{144x+xy^2}{2880y}$.
 5. 1 or 2 or $\frac{1}{2}$ or $-\frac{2}{3}; 3 \pm 2\sqrt{3}; -14 \text{ or } -19$. 6. $\frac{5}{6}\frac{8}{3}$.
 7. 2£. 8s. 8. 110 yds.

64.

1. $2+\frac{1}{x}$. 2. 5. 3. $-\frac{a^2(a+b)}{a^2+b^2}, abc$.
 4. 45, 55. 6. $7 \text{ or } 2\frac{2}{3}, \pm 2\sqrt{2}, \frac{4 \pm \sqrt{43}}{3}$.
 7. 29, 15, 11. 8. 8 miles.

214 TEST PAPERS IN ELEMENTARY ALGEBRA

65.

1. 2, 1. 3. $\frac{1}{x+y}, -\frac{6x+3}{(x+2)^2}$. 4. 2 metres.
6. -2; 3, -1; $\frac{6 \pm \sqrt{3}}{3}$. 7. 14 miles. 8. 42.

66.

1. 3, -6. 2. $\frac{17}{65}$. 4. $\frac{a^2}{16} - \frac{b^2}{4}$ sq. yds.
5. $\frac{3}{2}$, 37. 6. 45 m.p.h. 8. 60 sq. ins.

67.

2. $\frac{92x-79}{40}, \frac{1}{a^2+b^2}$. 3. -3. 4. 8.
5. 1 or $-\frac{1}{2}$; $\frac{1}{3}, \frac{1}{2}$. 6. 13. 7. 21. 8. 3 hrs.

68.

1. $-13x^2 - 11x - 2$. 2. $\frac{97}{30(x-5)}, \frac{x-2}{x+4}$. 3. 2, -5.
4. $6x+8y$. 5. 3, 5; 3 or -4 or -5; $\frac{-3 \pm \sqrt{65}}{4}$.
6. 15 miles. 8. 216 cu. ft.

69.

2. -4, 7. 3. $\frac{2}{(x-2)(x-4)}, \frac{a^2}{5(a^2-b^2)}$. 4. $\frac{1200xz}{1760x-y}$.
6. $\frac{1}{2}$; 7, 10, 9, 12. 7. 6 hrs. 8. 1500£.

70.

1. -7, -32, -25. 2. $4x^2 - 2x - 1, \frac{5x}{3(x^2-1)}$.
4. $2\frac{1}{2}$ m.p.h. 5. 4 or $-1\frac{1}{2}$; ± 2 or $\pm \frac{1}{2}$; 3, 2, 1.
6. 81. 7. $5\frac{1}{2}$. 8. 1s. 6d.

71.

1. $a^2+b^2+c^2, 0$. 2. x^3+4x+3 . 3. 24. 4. 5.
5. 2, 0 or 1 (repeated) or 4. 6. 56. 8. 9.

72.

1. $\frac{1}{a}$. 3. -7 , $-\frac{a^2}{b}$, 1 (repeated) or 3 or -1 .
4. $\frac{1}{a-b}[x(a^n - b^n) + a^{n+1} - b^{n+1} + 2ab^n - 2a^nb]$.
6. 1 m.p.h., 3 hrs. 8. $\frac{3}{10}$.

73.

1. $\frac{(x^2+1)^2}{(x+1)^4}$. 3. 0 or -11; -1, -2, -3; $\frac{3y+3}{2y+7}$. 4. 50.
5. x^2-3x-1 , $1+3x+6x^2+10x^3$. 6. 60.
7. $2x^2-7x+3$. 8. $4\frac{2}{3}$ m.p.h.

74.

2. $1-3x-6x^2+10x^3+21x^4+9x^5$, $1-5x+3x^2+9x^3$. 4. 15.
6. 4; 7, -4; $\frac{7 \pm \sqrt{61}}{6}$. 7. 27 miles. 8. 20, 50 m.p.h.

75.

1. $2x$. 2. $-\frac{1}{x}$. 4. 3, 12(...). 5. 50 brace.
6. 2 or $\frac{11}{14}$. 7. $-4\frac{1}{2}$. 8. 63 m.p.h.

76.

1. 2, 6, -6. 3. $\frac{a}{b}$. 4. $\frac{pq}{p+q}$.
5. -4; $\frac{1}{2}$ or $-\frac{2}{3}$; 3, -4. 6. 7. 8. 20, 12.

77.

2. $a = -2$, $x = -\frac{17}{3}$; $a = \frac{5}{6}$, $x = \frac{17}{18}$. 3. x , $\frac{7x-235}{42}$.
4. A has $\frac{3y+3z-x}{4}$ £. 5. -14 or -19 , $-\frac{a+b}{2}$.
6. 710 £. 7. $\frac{1}{10}$ or $\frac{2}{5}$. 8. 561 .

78.

2. $\frac{1}{x(x-1)^2}, \frac{x^2-9y^2}{x^2-16y^2}$. 3. 2. 4. $b; \pm 2$ or $\pm\sqrt{3}; \frac{3}{2}, 4$.
5. $1-6x+15x^2$. 6. 24. 7. 15, 16. 8. 20.

216 TEST PAPERS IN ELEMENTARY ALGEBRA

79.

1. 0.
2. 1 or 4, 5 or 2, 0 or $\frac{2}{3}$.
3. $x^2 + 3x - 7$.
4. $\frac{y - qx}{p - q}$ lbs.
5. 9.72 or $-.72$.
6. 35£.
7. 2 or $-1\frac{1}{2}$.
8. 48 min.

80.

1. $\frac{5}{3}(2x^2 - 4x + 5)$; $x = 4$, -2 , $y = -1$, 3.
2. $\frac{x+2}{y+2}$, $\frac{x}{(x+1)^2(x+2)^2}$.
4. $\frac{xyz}{x+y}$ miles, $z\left(\frac{x-y}{x+y}\right)^2$ hrs.
5. -3 , -5 , 6.
6. 35£.
8. -1 or ± 3 .

81.

2. 4.
3. 2, 3, -6 .
6. $\frac{1}{a} + \frac{1}{b}$; $x = 4$, $-\frac{1}{11}$, $y = -1$, $\frac{7}{11}$.
7. $\frac{xy}{y-x}$.
8. 100 miles.

82.

1. $\frac{ab}{ab+1}$.
2. $(b-c)(a-c)$.
3. $x^2 - 3x + 4$, $2x^6 - 3x^5 - 14x^4 + 23x^3 - 9x^2 + 28x - 12$.
4. $\frac{4}{a^2 - b^2}$, $\frac{2(x-1)(x+2)}{(x+1)(x+14)}$.
5. 1, 4.
7. 4.
8. 280 ft.

83.

2. $\frac{5y^2}{(x+3y)(x+2y)(x-y)}$, $\frac{3x-1}{x^2(x+1)^2}$.
4. ± 3 .
5. 13; 3 or $-\frac{13}{5}$, -5 or $\frac{31}{5}$; $a+b$.
6. 2 a penny, 3 for twopence.
7. 4, 6; $12x - 27$.
8. $CP = 2$ ins.

84.

1. -29 , -22 .
2. $\frac{5x^2 + 11x - 17}{x(x-1)(x-2)}$, $\frac{2ab}{a^2 - b^2}$.
3. -1 or $-\frac{1}{2}$.
4. $\frac{a+by}{x+y}$.
5. -4 , 2, 3; 3.
6. 20 miles.
7. -1 or $-\frac{3}{2}$, 1 or $\frac{3}{4}$.
8. ± 7 , ± 15 .

85.

1. $\frac{3}{2}, \frac{1}{2}$; 3775.
2. $4x^2 - 4xy - 3y^2, -2$.
3. $\frac{23+46x}{12(1-x^2)}, 4n^2-n$.
4. $\frac{4a^2+6ab-4b^2}{3a+b}$ pence.
5. $\frac{vf}{v-f}$; 2 or 1, 0 or -1.
6. 750£.
7. 5 ins.
8. 4, -5.

86.

1. $6(x-1)(x-2)$.
2. $\frac{a}{(a+b)(a-b)^2}$.
4. $\frac{a+b}{ab-a^2b^2}, x=y=1$ or 0.
5. 120 miles.
7. 4, 6 or 6, 4.
8. $\pm \frac{x^2-y^2}{8z} \pm \frac{z}{2}$.

87.

2. $\frac{1}{x-1}$.
3. 7, 2, -7.
4. $\frac{154880y}{3xz}$.
5. -1 or -3; 2 or -2, 3 or 6.
6. $\frac{22-5x}{x-3}$.
7. 120.
8. $\frac{1}{2}(3y-x)$ cwt.

88.

1. -1, -5.
2. $\frac{4x^3}{x^4-1}, y$.
3. $3z^2+y^2-3x^2$.
4. $\frac{b^2+ab+ac}{a+b}$.
5. $\frac{10}{x(x+2)}$.
6. 13 ft., 5 ft. 5 ins.
7. 3, 5.
8. 5 m.p.h.

89.

1. 0, $\frac{y}{x+y}$.
4. $\frac{1}{3}$.
5. -5, 7; 7.
6. 32.
7. 8 or $1\frac{1}{2}$; $\frac{2}{3}$ or $\frac{1}{7}$, $\frac{1}{4}$ or $-\frac{1}{7}$.
8. 2s.

90.

1. -3, 2.
2. $1-x^6, x^2+x^{2n+1}$.
4. $2; \pm\sqrt{2ab-b^2}; \frac{1}{2}$ or $\frac{3}{50}, \frac{3}{2}$ or $-\frac{79}{50}$.
5. 165.
8. 25 miles from B.

91.

1. $\frac{x^2+y^2}{2xy}, \frac{x^2y^2+1}{2xy}$.
2. $\frac{7}{5}, -\frac{8}{5}; 53; -192\frac{1}{2}$.
3. $2x-1$.
4. $-4, 2$.
6. $\frac{a+b}{2}; \frac{a}{b}$ or $\frac{b}{a}; x=z=\pm 6\sqrt{5}$ or $0, y=\pm 12$ or 0 .
7. 384£, 416£.
8. 10 m.p.h.

92.

1. 28 or 44, 44 or $-44, -56$ or 64 .
3. $5(x-y)(y-z)(z-x)(\Sigma x^2 - \Sigma yz)$.
5. $41y-6x$.
6. $-\frac{7}{4}$ or $0, \frac{7}{17}$ or $0; 1+a$ or $1-a$.
8. 100 miles.

93.

2. $\frac{x+2y}{x+3y}, \frac{113}{21(x-2)}$.
4. $3x^2-7x-2$.
5. $\frac{8}{3}, \frac{16}{3}; 2b, b$.
6. 9 m.p.h.
7. 1, 0.
8. 6.

94.

2. $\frac{4x^2+x}{4x-2}$.
3. $2x^2-x-3$.
4. $\frac{y-x}{(x+1)(y+1)}$.
5. 5, 2; a or a^3 .
6. 36 miles.
7. 10 or -2 ,
8. 126.

95.

2. $\frac{x+3}{x^4-1}, \frac{4(x^2+y^2)}{xy}$.
3. $\frac{1}{2}(x^2-7y^2-z^2)$.
4. $a; 2$ or $-\frac{4}{13}, 1$ or $-\frac{7}{13}$.
5. 22£.
6. 3, $-1, -1$.
7. 30 or 6.
8. 4 or $8\frac{4}{17}$ miles.

96.

1. $(x+1)(x-3)(3x^2-x+1)$.
4. 4.
5. 6, 9.
6. 12, 24.
7. $\frac{f(c)}{(c-a)(c-b)}$.
8. $1\frac{1}{2}$ hrs.

97.

1. $\frac{3x}{(x-1)(x^2+2)}$.
2. 69.
3. $x=y=1$ or -2 ; if $a=3, x=y=\text{any value}$; if $a=1$, any value satisfying $(x+\frac{1}{2})(y+\frac{1}{2})=\frac{9}{4}$.
4. x^2-2x+1 .
5. $\frac{1}{2}$.
6. 3 or -1 .
8. $\pm n, \pm(n-1), \pm 1$.

98.

1. $-5, 1$. 2. $x^2 - ax + b, x^3 - 2x - 3$. 3. $3x^2 + 3y^2 - 2xy$.
 4. $-2; 3, 3, -6; ab, a^2$. 5. 90 miles. 6. 5, 0, 5, 1.
 7. $\lambda = b, x = a$. 8. $a^2 - 2a - 1$ ft.

99.

2. $\frac{1}{4}$ or 3, $\frac{a}{b}$. 3. 3. 5. 0, $\frac{a^2 - ab + b^2}{ab}$.
 7. 8, 12. 8. $\frac{2qz(x+y)}{3p}$.

100.

1. $x + y - 3$. 2. $-3\frac{1}{2}; a, a + b$. 3. $\frac{3x}{100} \left(1 + \frac{x}{100} + \frac{x^2}{30000} \right)$.
 5. $\frac{1}{x^2}, \frac{b}{a}$. 6. 5, 23, 149. 7. 20 miles. 8. 12.48 p.m.

101.

2. 25 or -7 , -24 or $+24$. 3. $4:3; -1; \frac{9}{8}$.
 4. 30, 10. 5. 135 lbs.
 6. $x = y = 3$ or -1 (three times); 6 or -2 or $2 \pm \sqrt{-40}; \frac{c(a+b)}{1+c}$.
 8. 54 miles.

102.

1. $\frac{x}{8}$. 2. $\frac{1}{2}, -\frac{1}{2}, 1$. 4. 5, 2; $-b$; 0 or $\frac{1}{9}$.
 5. 144 ft. 6. $\frac{5}{32}, 2$. 7. 36. 8. 2 miles.

103.

1. $-2, 0, -5$. 2. $\frac{2y-3x}{5(x+y)}$. 3. $7\frac{1}{2}$.
 4. $\frac{2p+q}{3}; 3$ or $-3, 1$ or $7; \frac{5 \pm \sqrt{37}}{6}$. 5. $\frac{1}{2}$.
 6. $\pm 3, \pm 12$. 7. $13\frac{1}{2}$ lbs.

104.

2. $\frac{x}{x^2-1}$. 3. $3(\dots), 1(\dots)$. 4. $-2:13:8; 9$.
 6. 2 or 3. 7. $\frac{60(y-z)}{x-pz}$. 8. 7.

220 TEST PAPERS IN ELEMENTARY ALGEBRA

105.

- | | | |
|----------------------------|--------------|-----------------------------|
| 1. 0 or 4. | 4. -7. | 5. 300. |
| 6. $\frac{a}{y}(y+z)$ min. | 7. ± 6 . | 8. 15 miles from the start. |

106.

- | | | |
|--|--------------------------------|---|
| 1. $\frac{4x}{x^2+1}$. | 2. $2(x-1)(x-2) + (x-1) - 6$. | 5. 2 or $-\frac{1}{2}$; $\frac{ac}{b}$ or $\frac{bc}{a}$. |
| 6. $3a^3 + 3b^2 + 3c^2 - bc - ca - ab$. | 7. 16. | 8. 40 ft. per sec. |

107.

- | | | |
|----------------------------|---|-------------|
| 2. 3. | 4. $1\frac{1}{2}$, $-1\frac{1}{2}$, $\frac{1}{3}$. | 5. 54, -27. |
| 6. $\frac{y(pz-x)}{z-y}$. | 7. $1\frac{1}{3}$ hrs. | |

108.

- | | | |
|--|--------------------------------|---------|
| 1. 4. | 2. $x^3 + 2x^2 + 3x - 1$. | 3. 167. |
| 5. a or $\frac{b(a+b)}{a}$; 1 or 2 or -3, 1 or 2 or $\frac{1}{3}$. | | |
| 6. $\frac{4}{3}$, 3^{x-3y} . | 7. $\frac{pr}{p-q+r}$ o'clock. | |

109.

- | | | |
|--|--|------------------------------|
| 2. $3(2x^3 - 3x^2 - 11x + 6)$. | 3. 10, 24, 26 ins. | 4. 0, $-x$, $\frac{1}{y}$. |
| 5. 2 or 3 or 7 or -1, 3 or 2 or -1 or 7; 0 or $\pm\sqrt{-1}$. | | |
| 6. $\frac{2pq}{p+q}$ m.p.h. | 7. $x\left(2 + \frac{x}{100}\right)$ per cent. | 8. 35 min. |

110.

- | | | |
|--------------------------------------|--------------------------------------|--------------------------------|
| 2. $-\frac{1}{2}$, $2\frac{1}{2}$. | 4. 7 or 1, 5 or $-\frac{1}{2}$. | 5. 1 : -2 : -1 : 2. |
| 6. 95, 96 or -1, 0. | 7. $\frac{xy(z+1)}{xz+yz+x-y}$ days. | 8. $\frac{40000}{x^2} + 25x$. |

111.

- | | | |
|-----------------------------|--------------------|---------|
| 2. $\frac{2}{9}$. | 4. 2.57, .0000544. | 5. 2.2. |
| 6. 0, $\frac{y}{x^2-y^2}$. | 7. 8:5. | |

112.

1. $\frac{2x^2y^2}{x^2-y^2}$, 1. 2. $18(1-2x)$, $\frac{4}{3}$.
 3. 11; 1 or $\frac{1}{3}$, 1 or $\frac{5}{3}$; a or $-\frac{1+a}{a}$, a or $-\frac{1}{1+a}$.
 4. $10\frac{1}{2}\frac{0}{3}$. 5. 204, '699. 6. 2 hrs. 40 min.
 7. $-.8$, $.4$, $\frac{2a-4}{5}$, 2. 8. $\pm(b-c)$, $\pm(c-a)$, $\pm(a-b)$.

113.

1. 0 or $\frac{4}{3}(a^2+ab+b^2)$, $-\frac{1}{3}$. 4. '00168, '846.
 5. $\frac{y+z-2x}{3}$, $\frac{z+x-2y}{3}$. 6. 9·9. 7. 3 or $3\frac{1}{2}$ m.p.h.

114.

1. $24(\dots)$; $\left(\frac{3x+y}{2}\right)^2 - \left(\frac{x+5y}{2}\right)^2$. 2. $\left(\frac{y}{x}\right)^{x-y}$, q .
 3. '409, -195, 167. 4. $2[f(x)]^2-1$. 5. 8, 9, $x=-6$.
 6. 10, $10+7h+h^2$, $7+h$, 7 ft. per sec. 8. 1.45 p.m.

115.

2. 0; 12, 7; 2 or -1, $\frac{1}{2}$ or -1. 3. 8·56 sq. ins. 4. a .
 5. -2, 3. 6. 13. 7. 2, $-\frac{1}{3}$. 8. 15, 36, 39 ins.

116.

2. $\pm\frac{2}{3}$. 3. 7·4, 56·0. 4. $a=b=\frac{1}{2}$, 41075.
 5. 112. 7. 32. 8. 9d. a dozen.

117.

2. $\frac{a(a+b)}{b-a}$. 3. 248, 24·1. 4. $a=-b=\frac{1}{2}$.
 6. $\frac{(p+q)xy}{xy-x-y}$. 7. 254, 2000.

118.

1. $2(x^3+y^3)+x^2y+xy^2$. 2. 12, -5; 14 or -4. 4. 228, 2·31.
 5. $\frac{1}{4}$, $1-\frac{3a^2}{4}$. 6. $\frac{cpq}{aq+bp}$ hrs. 7. -14, $-\frac{ab}{a+b}$.

222 TEST PAPERS IN ELEMENTARY ALGEBRA

119.

- | | | |
|----------------|---|------------------------|
| 1. $x+y=1$. | 2. $\frac{1}{24}(x-1)(x-2)(x-3)(x-4)$. | |
| 3. $\cdot 3$. | 4. $45:4$. | 7. $3\frac{1}{2}$ hrs. |

120.

- | | | |
|---|---|----------------------|
| 1. $xyz=1$. | 2. $\frac{x(2x+y)(x^2-4y^2)}{y(y-5x)(x^2-y^2)}$. | |
| 3. 3 or -2 , -1 or $6\frac{1}{2}$; 3 or $\frac{-3 \pm \sqrt{13}}{2}$. | | |
| 4. $99\cdot 4$ cms. | 7. 47. | 8. $13\frac{8}{9}$. |

121.

- | | | |
|--|---|---|
| 2. 0 or $-6 \pm \sqrt{-11}$; 2, 3, 5; $\frac{cd(a+b)-ab(c+d)}{ab-cd}$. | | |
| 3. -1 . | 4. $6\frac{3}{4}$ miles. | 5. $\frac{x^{n-1}y}{(x+y)^m}$, $5-2\sqrt{3}$. |
| 6. $9:10$. | 7. $\frac{n^2-n+2}{2}$, $\frac{n(n^2+1)}{2}$. | |

122.

- | | | |
|---------------------------------------|------------------------------------|------------------|
| 1. $\left(\frac{a-b}{a+b}\right)^2$. | 3. $x=y=0$ or $a+b$; 2, 4, 6, 10. | 4. 15, 25 m.p.h. |
| 5. 2, 0, $19-20\sqrt{3}$. | 6. 96. | 7. 29£. |

123.

- | | | |
|---|--|--------------------------|
| 2. $\frac{4x^2-a^2}{x+a}$, $116\sqrt{2}$. | 4. $-\frac{2}{3}$. | 5. $\frac{n(5n-1)}{2}$. |
| 6. 150£. | 7. $\frac{1}{3}$, $-\frac{1}{2}$, $\frac{1}{6}$; 285. | |

124.

- | | | |
|---|---|--|
| 2. -2 or $\frac{2}{41}$, 3 or $-\frac{7}{41}$; $\frac{2a+b}{3}$. | 4. 7. | 6. $\frac{2}{\sqrt{x}}$ or $2\sqrt{x}$. |
| 7. $16\cdot 3$. | 8. $2\frac{3}{4}$ and $3\frac{3}{4}$ m.p.h. | |

125.

- | | | |
|-------------------------------------|-------------------------------|--|
| 1. 2. | 2. $\frac{1}{2}(x-1)(3x-2)$. | 3. $10\frac{5}{12}$, $4\frac{3}{5}$. |
| 4. 1·699, 1·255, ·176, 1·648, 70·0. | | 5. -27 . |
| 6. 2s., 4d. | 8. 48 runs. | |

126.

1. $\frac{x^4 - x^2y^2 + y^4}{x^2y^2}$, $\frac{a+b+c}{a-b-c}$. 2. $\frac{4}{5}$; 5 or 3, -3 or -5; $-\frac{1}{2}$.
 3. $3, \frac{n}{2}(3n+13)$. 4. $\pm\frac{10}{39}$, $\pm\frac{14}{39}$, $\pm\frac{35}{39}$. 6. 16.
 7. $\frac{1-\sqrt{1-e^2}}{e}$. 8. $20\frac{5}{8}$ miles.

127.

2. 3 or $-1\frac{1}{8}$; $\frac{c-a}{c+a}$, $\frac{a-c}{2(a+c)}$; 6. 3. 4. 4. 500.
 6. $\frac{1}{2}$, -2. 7. $1-x-x^2$, x . 8. 36 gallons.

128.

1. $(x-5)(x-7)$, $\frac{(x+y)^2+z^2}{z}$. 2. 14.
 4. 11 or $-17\frac{1}{3}$; $a+b$, $a-b$. 5. $180\sqrt{3}$. 7. 10. 8. 528.

129.

1. 4. 3. 0; $\frac{c}{a}(a-b)$. 4. 1, 4, 7, ...
 5. 14. 6. 12:5. 8. 10 a.m.

130.

2. $(ab-a-b-1)(ab+a-b+1)(b+1)(b-1)$. 4. 2.
 5. $(x^3+y^3)^{\frac{1}{3}}-y$. 6. 72 degrees. 8. 10 acres.

131.

2. $\frac{x(1+x^2)}{(1-x)^3}$, $\sqrt{37}+6$. 3. $\frac{5}{27}$; $\frac{2}{3}$, 2, $\frac{8}{3}$; ± 3 or $\pm\frac{3}{\sqrt{7}}$, ± 1 or $\mp\frac{5}{\sqrt{7}}$.
 4. 20, 16. 5. $2^{\frac{n^2+n}{2}}-2^{\frac{n^2-n}{2}}$. 6. $11\frac{3}{7}$ miles from X.
 7. 10. 8. $-\frac{1}{2}$, $\frac{1}{2}$, $-\frac{1}{2}$.

132.

2. 1265, $\frac{401}{15}$, $\frac{424}{15}$. 3. 3; ± 3 or $\pm\frac{1}{\sqrt{3}}$, ∓ 1 , $\pm\frac{4}{\sqrt{3}}$; 6, 4 or 9, 9 or 4.
 4. $\frac{y}{2x+2z}$ m.p.h. 6. 99, 121.
 7. 1, 1, 2, -1. 8. $41\frac{7}{13}$ min. past three.

224 TEST PAPERS IN ELEMENTARY ALGEBRA

133.

2. 144, 89; 3; 0 or ± 3 , ± 2 or ± 1 . 3. 17, 2.
5. 18. 6. 11. 7. $\frac{2}{3}$, $\frac{1}{3}$. 8. 11:24.

134.

2. $\frac{1}{2}$, ± 1 . 3. $\sqrt{3} - \sqrt{2}$, 1. 5. $2x^2 - 9y = 36$.
6. 60 or 40. 7. 291. 8. 5 min.

135.

2. $\frac{a+b}{a-b}$. 3. $2\frac{1}{2}$, $-2\frac{1}{2}$, 3. 6. $4(a+b)$. 8. $\frac{mh}{m+2}$.

136.

1. $\frac{b^2}{a^2} - \frac{a^2}{b^2}$; $\frac{x^2 - xy + y^2}{(x-y)(x^2 + y^2)}$.
4. ± 3 or $\pm\sqrt{2}$, ± 2 or $\mp 4\sqrt{2}$; a or $-\frac{a+1}{a}$, a or $-\frac{1}{a+1}$.
5. $P r^n - \frac{A(r^n - 1)}{r - 1}$. 6. 200000. 7. $5\frac{1}{2}$, $6\frac{3}{5}$ hrs. 8. $\frac{l}{2}$.

137.

2. $\frac{2a}{b}$, $-\frac{b^2}{a^2}$. 3. -16. 4. 8 or -1. 7. 13, 32, 11 lbs.

138.

2. -2, $\pm\sqrt{2}$. 3. 3, 1; $\pm \frac{ac}{\sqrt{a^2 + b^2}}$, $\mp \frac{bc}{\sqrt{a^2 + b^2}}$.
4. $\frac{1}{p-q} [(p-r)v + (r-q)u]$. 6. 20 m.p.h.
7. ± 10 , ± 16 . 8. 1s. 5d.

139.

2. ± 9 , ± 1 ; 3. 3. $\frac{2(a^2 + ab + b^2)}{a+b}$, 3^{n-1} .
7. 8 lbs. $10\frac{8}{9}$ ozs. 8. 300.

140.

1. 3, -3, 3. 4. ± 4 or $\pm \frac{7\sqrt{3}}{2}$, ± 3 or $\mp \frac{5\sqrt{3}}{6}$; $\frac{(a-b)^2}{4b}$.
5. 14 units of area. 6. 1, 5, 7. 8. $\frac{a}{2}$, $2a$ ft.

141.

2. $\frac{3f(x) + [f(x)]^3}{1 + 3[f(x)]^2}$. 3. $2^n + n^2 + n - 1$.
4. ± 6 or $\pm\sqrt{-5}$, ± 5 or $\mp 4\sqrt{-1}$, ± 4 or $\pm 5\sqrt{-1}$;
 0 or $a+b$ or $\frac{a^2+b^2}{a+b}$.
5. 0 or 4, 14. 6. 15 m.p.h.

142.

2. 1. 3. 52. 4. $-(\Sigma xy)(\dots)(\dots)(\dots)$. 5. 4:1.
6. $\frac{3}{4}$, $a=10$. 7. 6.1 ft. per sec. 8. $\alpha(3x+y-5)$.

143.

2. $x=\frac{3}{2}$, $y=\frac{1}{2}$ or any solution of $2x+y=2$. 4. 5 or $-\frac{1}{3}$.
5. 7. 6. 1 or 5; 2 or ± 3 , ± 3 or 2.
7. 5:2:3. 8. 2, 1; $3^{n+1} + \frac{1}{2}(n+3)(n-2)$.

144.

1. $\frac{x-y+1}{x-y-1}$, $\frac{4x}{1-x^2}$. 2. 3; $\frac{3}{2}$, $\frac{3}{2}$.
3. $\frac{2.9}{4}$. 6. 78.5. 8. 45, 80£.

145.

3. $x^2 - 7x - 1 = 0$, $x^2 - 6x - 44 = 0$. 4. $\frac{np}{mn - m - 1}$.
7. 7, $7+h$, 7 ft. per sec. 8. $1\frac{1}{3}$ hrs.

146.

2. 5. 4. $x^4 - 5x^3 + 5x^2 - x = 0$; $2p^2 + q = 0$.
5. 1, -1; 4, 5, 6. 6. 144.

147.

1. $15 \cdot 2^{2n+1}$. 2. $6p^2 = 25q$, 4. 3. $(3y+x)(2y-x) = 25$.
4. 19. 5. 0. 6. 9, 40, 41. 7. $\frac{1}{a-b}$, $\frac{1}{a-b}$. 8. 10.

148.

1. $x - x^{-1}$. 5. a^2 or $2ab - a^2 - 2c$; 2 or $-\frac{10}{13}$, 3 or $\frac{15}{13}$.
6. -6, 6. 8. 1s. 6d.

149.

1. 130, 1·25, 7420. 2. $\frac{19}{5}$, 5; $\frac{b^2}{a}$ or $-\frac{1}{a}$. 4. -1.
 5. 2, -5; 177; $6n^2 - 6n - 3$. 6. 3 or $-\frac{4}{3}$.
 7. $\frac{1}{2}n(n-1) + m(m+2n-1)$. 8. $\frac{b^3x}{a^3n}$ min.

150.

1. ·135, 18·3. 3. 21. 4. 494550.
 6. $\frac{30}{n}(1 + \sqrt{1+4n^2})$ m.p.h. 8. 3.

151.

1. 2, $-\frac{p^2}{q^2}$. 2. $x+2y$. 4. $\frac{a^3+b^3}{2ab}$, $\frac{a^3-b^3}{2ab}$; 9. 6. 484 sq. ft.
 7. 720, 120. 8. $23h+5h^2$ ft.; $23+5h$, 23 feet per sec.

152.

2. $\frac{a\sqrt{a}}{\sqrt{a}\pm\sqrt{b}}$, $\frac{b\sqrt{b}}{\sqrt{b}\pm\sqrt{a}}$; 7, 2. 5. 40 miles.
 7. 1326, $\frac{1}{26}$. 8. $\frac{2x-9}{57}$, $\frac{2x-13}{57}$.

153.

3. $1\frac{1}{4}$; $\pm\frac{3}{4}$, $\pm\frac{2}{3}$. 4. 10080. 5. $9x^2 - 45x + 11 = 0$.
 7. 4£. 17s. 6d., 2£. 12s. 6d., 1£. 10s. 8. $\frac{2abx}{a^2-b^2}$.

154.

2. 2^{3n-7} . 3. 3 or -5; $a^2+b^2+16ab=0$.
 4. 144. 6. $\frac{61}{48}$. 7. $h = \frac{m}{a-b}$, $n = \frac{ma}{a-b}$.
 8. $(10a-7b)^2 - (7a+2b)^2$; $(26a-9b)^2 - (25a-6b)^2$.

155.

1. $4x^2+2$. 3. 3600, 1440. 5. $2\frac{3}{4}$. 6. 48, 49, 50. 8. 2160 lbs.

156.

3. ± 8 or $\pm \frac{17}{3}\sqrt{\frac{2}{7}}$, ± 3 or $\pm \frac{26}{3}\sqrt{\frac{2}{7}}$; $\frac{cd(a+b)-ab(c+d)}{ab-cd}$. 4. 12.
 5. $3(2^{n+1}-2)+\frac{1}{2}(5n^2+n)$. 6. $x^2-24x+120=0$.
 7. $\frac{p(y-z)+q(z-x)}{y-x}$ min. past 12, $\frac{k(y-x)}{q-p}$ m.p.h. 8. $x^2+y^2=\frac{1}{2}a^2$.

157.

2. 0; a or $\frac{c^2}{a}$; 1, $1\frac{1}{2}$, 2. 4. $x^2-2hx+2h^2=1$, $h=\pm 1$.
 5. 4320, 14400. 8. After $3\frac{2}{3}$ miles.

158.

1. $3x^2+4y^2+9z^2$. 2. $x-1$, $(x-1)(x^2+2x+2)(x^3+2x^2+2x+1)$.
 4. 9 or 16. 6. 576. 7. $\frac{60(x-y)}{kx+60}$ m.p.h.

159.

1. 117 lbs. 3. $6x^2+6y^2+6z^2-7xy-7yz-7zx$.
 4. $2(n-3)(n-4)\underline{n-3}$. 7. $\frac{2b(a+c)(a-b)(c-b)}{(a-2b+c)^2}$.

160.

2. 0 or $-\frac{2}{7}$; 0. 3. $\frac{1}{2}n(n-1)$. 4. $\frac{3}{2}$.
 6. $\frac{a+b}{2}$. 7. 132 yds. 8. $\frac{57}{43}$.

161.

2. $-3\frac{1}{2}$, $-4\frac{1}{2}$, 9; 5, -5. 5. 24 sq. ins., 4 cu. ins.
 6. $\frac{1}{x+1}-\frac{2}{(x+1)^2}$; $-\frac{1}{4(x-1)}+\frac{1}{4(x+1)}+\frac{3}{2(x+1)^2}$. 7. 1, $\frac{1}{2}$.

162.

2. 3, 6; 63, 33, 1. 3. $\frac{3}{5(x-2)^2}+\frac{22}{25(x-2)}-\frac{3}{4(x-1)}-\frac{13}{100(x+3)}$.
 4. a or 0; $x=y=a$, $z=\frac{b^2}{a}-a$, etc. 5. 20, 28 hrs.
 7. 126, $\frac{4}{9}$; 144. 8. $\frac{a^{1+\frac{1}{n}}-b^{1+\frac{1}{n}}}{a^{\frac{1}{n}}-b^{\frac{1}{n}}}$, $\frac{10}{81}(10^n-1)-\frac{n}{9}$.

228 TEST PAPERS IN ELEMENTARY ALGEBRA

163.

1. x^2+1 .
2. $x^2+2x+2=0$.
3. 22100, 4804, 1275.
4. $\frac{3}{x+1}-\frac{5}{4x-1}$.
5. 1, 7, 13, $6n-5$.
6. $\pm\sqrt{7}$; 3 or -1, 5 or -3, 7 or -5.
7. 33, 42 m.p.h.
8. $\frac{1}{2}$.

164.

1. $8\left(x+\frac{1}{x}\right)$.
2. 4 or $-1\frac{2}{3}$, $\frac{25a}{24}$.
3. $\frac{2}{(x-1)^2}+\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$.
5. $a=b$ or $a^2+ab+b^2=0$.
6. $\frac{5}{16}$.
7. 1£.
8. $\frac{xy}{2x-y}$ min.

165.

2. $\frac{2}{1-x}+\frac{3}{2-x}-\frac{6}{(2-x)^2}$.
4. $3^{n-2}(q-p)$.
5. $-\frac{1}{3}$.
6. $pr+(2q+r)(q+r)=0$.
7. $\frac{7}{9}$.
8. 429.

166.

1. 47·85.
3. $x^2+2+\frac{1}{x-1}-\frac{1}{x+1}$.
4. $30(7a-5b)$; $2^{20}+419$.
5. $\frac{15\pm\sqrt{85}}{14}$; 4 or 7, -3 or -5.
6. $1\frac{2}{3}$ m.p.h.
7. $\frac{28}{51}$.
8. $a+b=c$.

167.

2. a or b ; 1, 1.
3. 269400.
4. 1, $-\frac{x^ay^b}{x^a+y^b}$.
5. $\frac{3}{4}$, $-\frac{1}{4}$, $\frac{1}{4}$.
7. 8d.
8. $\frac{1}{6}n(n-1)(n-2)$.

168.

1. $x=3$, $y=4\cdot6$; $a=3$, $b=4$, $c=7$.
2. $\frac{c^2x^3}{cx-b}$.
4. 1680.
5. 20.
6. $3\frac{2}{11}$, $9\frac{6}{11}$; 3.
7. 15 m.p.h.
8. 1892 A.D.

169.

2. $\frac{1}{(a-b)^2} \left[\frac{f(b)}{x-b} + \frac{(a-b)f(a)}{(x-a)^2} - \frac{f(b)}{x-a} \right].$ 3. 267148.
 6. $x^2 - 5x + 2 = 0.$ 7. 2300, 2860 sq. yds.

170.

2. $\frac{28}{x-3} - \frac{28}{x-2} - \frac{26}{(x-2)^2} - \frac{15}{(x-2)^3}.$ 3. 55.
 4. $3(\dots)(\dots)(\dots).$ 5. $\pm 4, \pm 5; b-a.$ 7. $\frac{8}{27}.$ 8. $17\frac{1}{2}.$

171.

2. $-5\frac{1}{2}, 3\frac{1}{4}.$
 4. 2 or 3 or $2\frac{1}{2}; a^2(c-b), b^2(a-c), c^2(b-a); \frac{-5 \pm \sqrt{2}}{2}.$
 6. 50, 75 min. 7. 12.

172.

2. $(b+d)(b-d)(a+b+c-d)(a+b-c+d); 12(\dots).$
 3. $\frac{1 \pm \sqrt{1+4a^3}}{2a}, \frac{1 \mp \sqrt{1+4a^3}}{2a}; -6, -9, \frac{-15 \pm \sqrt{41}}{2}; 2 \text{ or } -1.$
 5. $x^2 - 21x + 4 = 0; q^3 - p^3v = 0.$ 6. $9 - 14n.$
 7. $-\frac{1}{7}.$ 8. $3\frac{1}{2}\frac{3}{9}$ miles.

173.

3. $\pm \frac{a}{2}\sqrt{5}; \cdot 6, \cdot 4.$ 4. 1. 5. 6. 8. 9.

174.

1. $x^{\frac{1}{2}}y^{\frac{1}{4}} - 2x^{\frac{1}{3}}y^{\frac{1}{2}} + 2x^{\frac{1}{6}}y^{\frac{3}{4}}; 3+a+b+3ab.$
 4. $-6 \text{ or } -8 \text{ or } -7 \pm \sqrt{19}; \pm 1 \text{ or } \pm 2, \mp 2 \text{ or } \pm 1.$
 6. $\frac{q(1-p)}{p+q-2pq}.$ 7. 100 or 50 ft.

175.

2. 2, 3, 5; 0 or $\pm\sqrt{-1}$ or $\pm\sqrt{3}$, 0 or $\pm\sqrt{-1}$ or $\mp\sqrt{3}.$
 4. $n^4.$ 7. 4725.

230 TEST PAPERS IN ELEMENTARY ALGEBRA

176.

3. $\frac{1}{2}\left(\frac{a^4}{b^2} + \frac{b^4}{a^2}\right), \frac{1}{2}\left(\frac{a^4}{b^2} - \frac{b^4}{a^2}\right); \frac{a+b}{2}.$ 6. 85, 65 grs.
7. $BP=4$ ins. 8. 36.

177.

3. $\pm 1, \pm 2; 2$ or $-1.$ 4. 60. 5. $-3.$ 7. $(l+2m+n)^2=4nl$

178.

4. $3 \cdot 2^n + 2n^2 - 5n - 3.$
6. $\pm 3, \mp 2; a$ or $\frac{a^3 - 3ab^2}{a^2 + b^2}, -b$ or $\frac{3a^2b - b^3}{a^2 + b^2}.$ 7. 17420. 8. 15.

179.

1. $26\frac{2}{7}.$
4. 2 or 16, 2 or $\frac{1}{2}; 3k, 7k, 11k, 20k,$ where $k=1$ or $\frac{-1 \pm \sqrt{-3}}{2}.$
7. $11\frac{1}{4}$ m.p.h. 8. $AP=3$ ins.

180.

2. $\frac{1}{2(x-1)^3} + \frac{1}{2(x-1)^2} - \frac{1}{4(x-1)} + \frac{x-1}{4(x^2+1)}.$ 5. 1, $-2.$
6. $x^{-\frac{47}{6}}.$ 7. $\frac{1}{a}, \frac{1}{b}.$ 8. 2:3:3.

181.

3. $-16(\dots); (ab-a-b-1)(ab+a-b+1)(b+1)(b-1); (a+b)(b-c)(c-a).$
5. 3 or -1 or $\frac{3}{2}(-1 \pm \sqrt{-3})$ or $\frac{1}{2}(1 \pm \sqrt{-3}),$
 1 or -3 or $\frac{1}{2}(-1 \pm \sqrt{-3})$ or $\frac{3}{2}(1 \pm \sqrt{-3}); 1$ or $\frac{c(a-b)}{a(b-c)}.$
6. $\pm 3.$ 7. $-\frac{4}{5}, -\frac{7}{5}.$
8. 4 days, total cost varies as square root of daily wage.

182.

1. $2x^2 + 2y^2 + 2z^2 + 3xy + 3yz + 3zx.$ 3. 2, $-1.$ 4. 4, 7, 10. 5. $\frac{1}{2}.$

183.

1. 26. 5. $3^r(2r-1).$ 7. $\frac{44}{4165}.$ 8. $43\frac{1}{3}$ m.p.h.

184.

1. $(x-y)(x+y)^2$.

2. $x=a, y=b, z=-b$, etc.

5. $25c^2 < 49$.

8. $\frac{a(b+c)}{3bc-b^2}, \frac{a(b-c)}{3bc-b^2}$ m.p.h.

185.

1. $x^2+y^2+z^2-yz-zx-xy$.

2. $x > \frac{1}{3}$ or $< -\frac{3}{2}$.

3. $\frac{1}{4}(\sqrt{6}+2\sqrt{3}-2-\sqrt{14})$.

5. $2 \pm \sqrt{2}, 2 \mp \sqrt{2}; \frac{1}{2}$. 8. 5:3.

186.

2. $3x^2-15x+17=0$.

5. $-\frac{1}{x+1} + \frac{3}{x-1} + \frac{2x-4}{x^2+1}$.

6. $p = \frac{f(a)}{(a-b)(a-c)}$.

8. 520.

187.

1. 10, 0.

5. 1 or -3, 2 or -4, 3 or -5.

7. $y^5-6y^4+7y^3+7y^2-6y+1=0$.

8. 69 days.

188.

2. $\frac{1}{4}$. 3. $\frac{1}{3}, 4, 5$. 4. 4 or -2, 3 or -1, 2 or 0; 3 or 20, 4 or -30.

5. 6 per cent.

8. 9:6:4 or 1:1:1.

189.

1. $2-\sqrt{2}$.

2. $-4 \pm 2\sqrt{6}$.

3. $\pm 1, 1; \frac{5}{4}, \frac{17}{4}$.

5. 2278000.

8. 6 cwt.

190.

4. 3 or -2, 2 or -3; 25 or 9, 9 or 25, 15.

5. $\frac{1}{2}a^2(b+c) + \frac{1}{2}b^2(c+a) + \frac{1}{2}c^2(a+b)$.

6. $-\frac{6}{7}, -\frac{2}{7}$.

7. $(p^2-qr):(q^2-rp):(r^2-pq)$.

8. 9.30 a.m.

191.

1. $x(x+3)(x-3)(x^2+3x+9)$.

3. 11.

5. -3 or 2 or $-2 \pm \sqrt{2}$, 3 or 3 or $-1 \pm \sqrt{2}$; $x=1, y=2$ or $-3, x=2, y=1$ or $-3, x=-3, y=1$ or 2 .

8. 2541 and 5412.

232 TEST PAPERS IN ELEMENTARY ALGEBRA

192.

4. ${}^nC_r(3^{n-r} - 2^{n-r})$. 8. 17, 51.

193.

3. 1680. 5. 1 or $\frac{2}{5}$, $\frac{1}{2}$ or $\frac{2}{3}$; $1 + \sqrt{-1}$ or $1 - \frac{3}{2}\sqrt{-1}$. 8. $4n - 5r = 3$.

194.

5. $\frac{1}{2a^2} \left(\frac{1}{a-x} + \frac{2a}{(a-x)^2} + \frac{x-a}{x^2+a^2} \right)$.
6. $x^3 + 2ax^2 + 8b^2 = 0$. 8. $p + q - 2pq$.

195.

4. 27 or $\frac{75}{441}$; cd or $\frac{1}{3}(2c^2 + cd + 2d^2)$. 7. 153900£.

196.

4. 1, 2, 3, 4. 6. $\frac{5}{8}$ or $\frac{3}{8}$ or $\pm \frac{7}{8}$, $-\frac{3}{8}$ or $-\frac{5}{8}$ or $\pm \frac{7}{8}$; $-a$.

197.

5. ± 2 or $\pm \frac{1}{2}$, $\mp \frac{1}{2}$ or ∓ 2 ; $x = y = 0$ or $\pm \frac{\sqrt{21}}{2}$, $x = -y = \pm \frac{\sqrt{13}}{2}$; ± 3 , ± 2 , ± 7 .
7. 2:1:-2. 8. 40 sheep, 8 oxen.

198.

4. $p + q - r + 1$. 6. 9 or 25, 25 or 9, 15; 9, -8.

199.

2. $\frac{2}{1-x} + \frac{3}{(2-x)^2}$. 5. $\frac{2\sqrt{3} - \sqrt{6}}{6}$.
6. ± 2 , ± 3 ; 0, c , $\frac{c}{2} \pm \sqrt{\frac{c^2}{4} - \frac{a^2 + ab + b^2}{a+b}}$.

200.

2. α , $\frac{\alpha}{2}[b - 1 \pm \sqrt{b^2 - 2b - 3}]$; $\frac{2^{2m+1} - 1}{2^{2m+1} + 1}$. 6. 138000 gallons.

GRAPHICAL EXAMPLES.

1. 9. 2. 28. 3. 1·5, 2·5. 4. 2·65, ·57.
5. 2·19 or - 3·19. 6. 1 or ·5; 1·78 or - ·28; 1·31 or ·19.
7. 2·13 or - ·47, ·42 or - 1·31. 8. - ·12. 9. ·62 or - ·18.
10. ·6, 4. 11. 1 or - 4. 12. 12. 13. 7·61 or ·39; 16.
14. 1·2. 15. 4, 2·5, 7·8. 16. 24 m.p.h., 7·7 miles from *A*.
17. 1 or $-\frac{1}{2}$. 18. 4·7, - ·5, - 4·2. 19. 3·28, 4·63. 20. 8·51 p.m.
21. *A* rides 8 miles, then walks; *B* rides 4 miles and walks to *A*'s bicycle; $1\frac{2}{3}$ hrs.; yes.
22. 2 or - ·81, 3 or - ·75. 23. 61.
24. 5·24 or ·76; 1 or 3; 1 or 6; 6·25.
25. 3·45, 4·21, 4·46 p.m.; 3·58 p.m.; 3·38, 4·14 p.m. 26. 50 miles.
28. 9·35 miles from *A*. 29. 1 or 4 or 7, 6 or 4 or 2.
30. 2·49, ·29, - 2·78. 31. $2·74 > x > - 3·74$.
33. 308 sq. ft. 35. 10·5 millions, ·37 million per year.
37. $xy = 11·4x + 44$. 38. 14·6 and 28·1.
39. 1·76 or 10·24. 40. 2·5 cms.

WALSLEY HIGH SCHOOL

BY CLEMENT V. DURELL, M.A.

A Course of Plane Geometry for Advanced Students

8vo.

PART I. 5s. net. PART II. 7s. 6d. net.

SOME PRESS OPINIONS.

PART I.

Journal of Education.—"Mr. Durell has supplied a distinct want and supplied it well. . . . The book is one of many merits, not the least of which is the low price at which it is published."

Mathematical Gazette.—"It deserves to be widely known; we wish it every success, and hope that it will become generally adopted, for it is a good book. Instead of a mass of bewildering details without method or connecting thread, such as we have become accustomed to expect in English text-books on the earlier parts of Geometry, we are given the essence of what is useful for further progress. . . . The book is singularly successful in bringing out the principles of Geometry; and it forces the reader to think, although it is seldom hard to understand. It is also remarkably complete, nothing essential being omitted, and very little that is unessential being admitted."

PART II.

School World.—"The work will be found most stimulating and useful. The historical notes add greatly to the interest; the general arrangement is admirable, and there is a valuable collection of examples."

Educational Times.—"The treatment is interesting, and provides material for an excellent course of study. . . . The gathering up of the threads of history, which have been collected and which are here presented to the reader in the form of observations preliminary to the discussion of fresh facts, creates one of the most attractive features of the book and infuses life and charm into its pages."

Nature.—"A trustworthy, practical, and interesting text-book."

LONDON: MACMILLAN & CO., LTD.

:

BY H. S. HALL, M.A., AND F. H. STEVENS, M.A.

A SCHOOL GEOMETRY

Crown 8vo.

PARTS I. AND II.—*Part I.* Lines and Angles. Rectilineal Figures.
Part II. Areas of Rectilineal Figures. Containing the substance of Euclid Book I. 1s. 6d. Key, 3s. 6d.

PART I.—Separately. 1s.

PART II.—Separately. 6d.

PART III.—Circles. Containing the substance of Euclid Book III. 1-34, and part of Book IV. 1s.

PARTS I., II., AND III. in one volume. 2s. 6d.

PART IV.—Squares and Rectangles. Geometrical equivalents of Certain Algebraical Formulæ. Containing the substance of Euclid Book II., and Book III. 35-37. 6d.

PARTS III. AND IV. in one volume. 1s. 6d.

PARTS I.-IV. in one volume. 3s. Key, 6s.

PART V.—Containing the substance of Euclid Book VI. 1s. 6d.

PARTS III., IV., AND V. in one volume. 2s. 6d.

PARTS IV. AND V. in one volume. 2s.

PARTS I.-V. in one volume. 4s.

PART VI.—Containing the substance of Euclid Book XI. 1-21, together with Theorems relating to the Surfaces and Volumes of the simpler Solid Figures. 1s. 6d.

PARTS IV., V., AND VI. in one volume. 2s. 6d.

Key to Parts V. and VI. 3s. 6d.

PARTS I.-VI. in one volume. 4s. 6d. Key, 8s. 6d.

LESSONS IN EXPERIMENTAL AND PRACTICAL GEOMETRY. Cr. 8vo. 1s. 6d.

A SCHOOL GEOMETRY. PARTS I. AND II. With an Introductory Course of Experimental and Practical Work. Cr. 8vo. 2s. 6d.

SOME PRESS OPINIONS.

Journal of Education.—"Seems quite the best of the new text-books of geometry that have lately been published. It is moderate in its changes and excellent as to arrangement and selection. There are numerous exercises, graphical and theoretical. This new work should satisfactorily replace the former excellent Euclid by the same authors, which till now has been used by so many schools."

Saturday Review.—"Admirable, and thoroughly well done."

Guardian.—"The completed work takes rank among the very foremost of modern text-books, both for the admirable structure of its component parts as well as for the soundness and moderation of its changes."

LONDON: MACMILLAN & CO., LTD.

R.B.A.N.M'S H.S (M) LIBRARY

B'lore-42

Accession No ;

312

U.D.C. No :

512/DUR)

N114

22.1.1987

Date;

18 SEP 1986

13 NOV 1986

R. B. A. N. M'S
High School (Main) Library

BOOK CARD

Author.....DURELL (CLEMMENT)

Title.....Test papers in E
.....ary Algebra.

Account No. 312.

Class No 512 (DUI)

Name of the Borrower

Ca

Surya Prakash
Rajgopal

